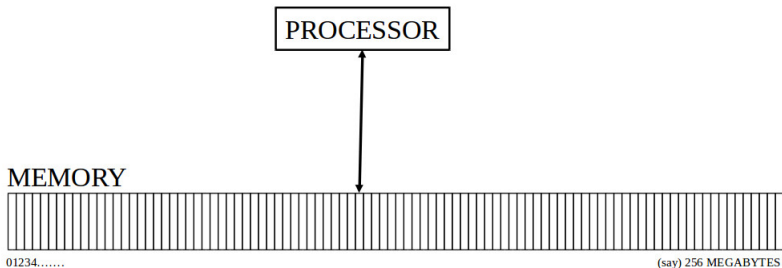


CS1100 – Introduction to Programming

Lecture 3

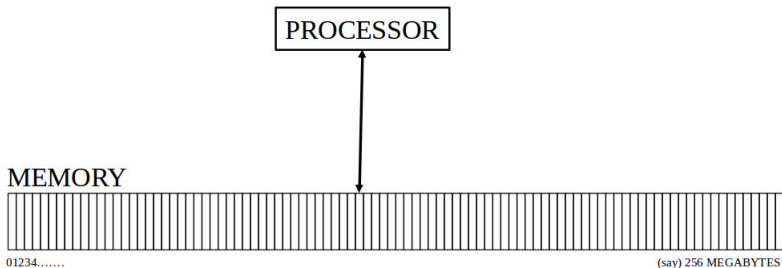
Instructor: Shweta Agrawal ([shweta.a@cse.iitm.ac.in](mailto:shweta.a@cse.iitm.ac.in))

# The Computing Machine



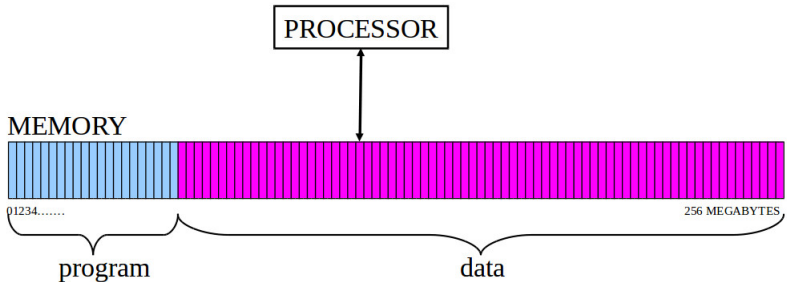
- The computer is made up of a processor and a memory.

# The Computing Machine



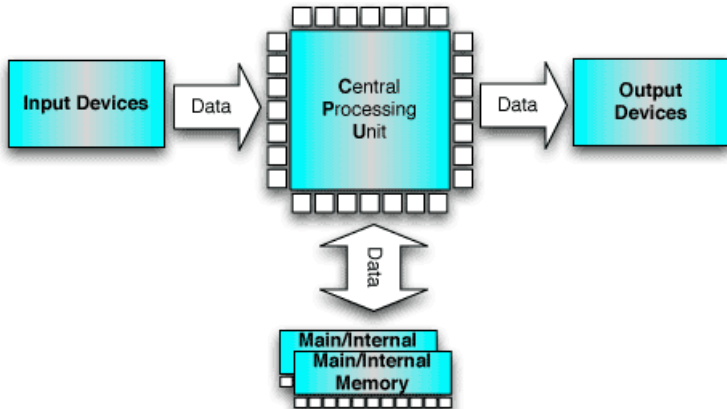
- The computer is made up of a processor and a memory.
- The memory can be thought of as a series of locations to store information.

# The Computing Machine



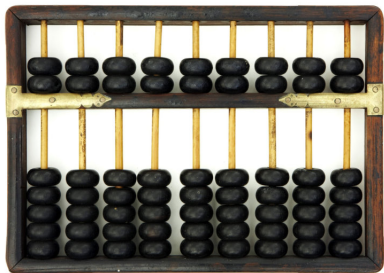
- A program is a sequence of instructions assembled for some given task.
- Most instructions operate on data.
- Some instructions control the flow of the operations.

# The Computing Machine : von Neuman Architecture



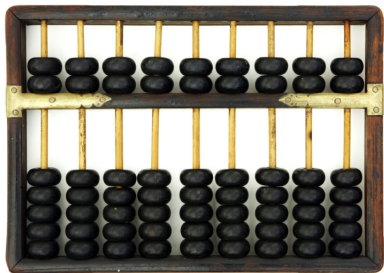
A brief look into the history...

## From Abacus to Apple



- Counting frame.
- One of the earliest form of calculator.
- Still used by kids to do fast simple arithmetic.

# From Abacus to Apple



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  - One of the earliest form of calculator.
  - Still used by kids to do fast simple arithmetic.
- 
- Followed by mechanical calculators by B. Pascal (1642), G. W. Leibniz (1671).
    - Used cogs / interlocking gears.
    - Performed  $+$ ,  $-$ ,  $*$ ,  $/$ ,  $\sqrt{\quad}$ .
    - Leibniz is credited of creating the binary system.



## Jaquard looms (1804)

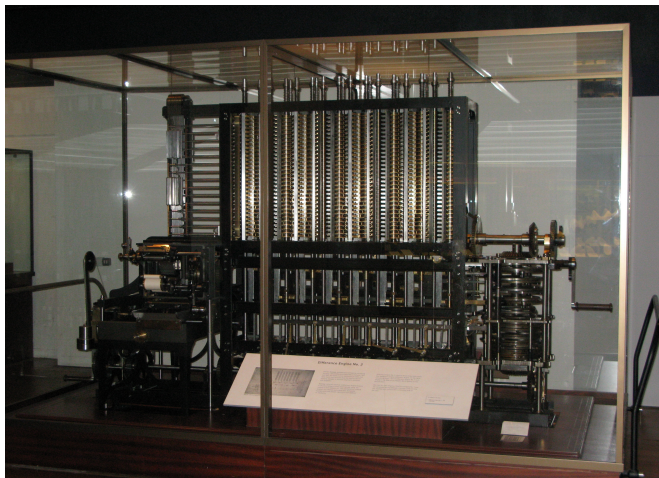


## Charles Babbage (1791–1871)



- Regarded as the “Father of Computer” .
- Conceived of a machine that has all the parts of a modern computer, input, a memory, a processor, and an output (1850).

## Difference Engine (1850)



Difference engine built from Babbage's design  
(London Science Museum).

## Ada Lovlace (1815–1852)



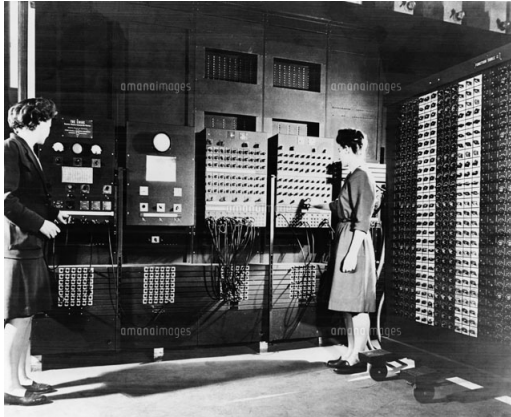
- “Wrote” the description of the mechanical computer of Babbage.
- Regarded as the first programmer ever.
- The programming language ADA is named after her.

## Alan Turing (1912 – 1954)



- Father of Theoretical Computer Science (TCS) and Artificial Intelligence (AI).
- Turing machine – a model for a general purpose computer.
- Turing test – how intelligent is a machine?

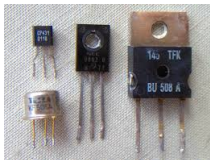
# First Electronic Computer : ENIAC 1946



Electronic Numerical Integerator  
And Calculator.

- 50,000 vacuum tubes, diodes, relays, resistors, capacitors.
- 5 million hand-soldered joints.
- Weighed 27 tons.
- Covered  $167m^2$  area.
- Consumed 150 kW of power.

1946 – 1976

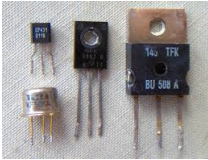


Transistors



Integrated Circuits

1946 – 1976



Transistors



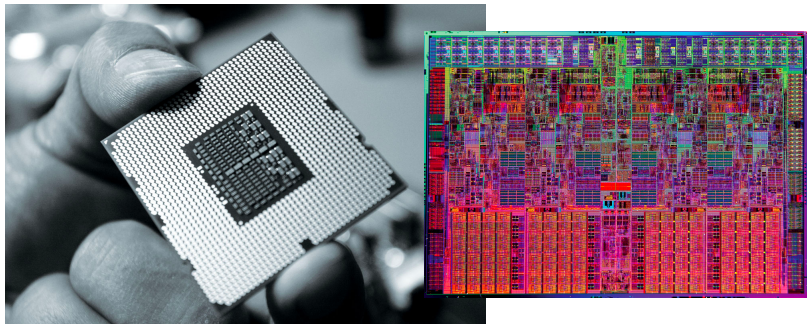
Integrated Circuits



Apple Macintosh



## Today's World : Core i7 Processor



2008-15: Intel Core i7 Processor

Clock speed:  $> 2.5$  GHz

No. of Transistors:  $0.731 - 1.3B$

Doubles every two years (Moore's law!)

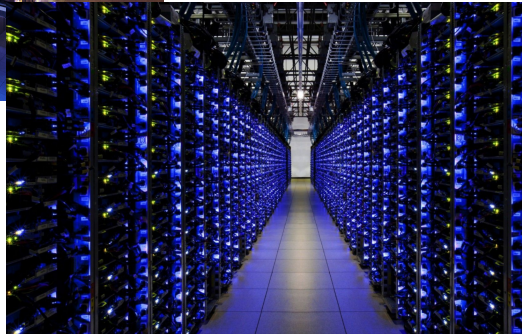
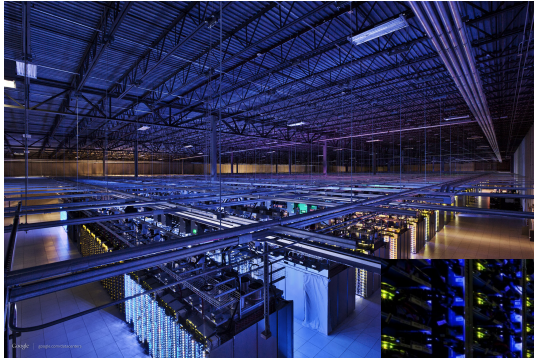
Technology:  $45 - 22nm$  CMOS Area:  $263 - 181mm^2$ .

Nowadays: Multicore (as clock speed increased) with cooling units!

# Modern computing devices



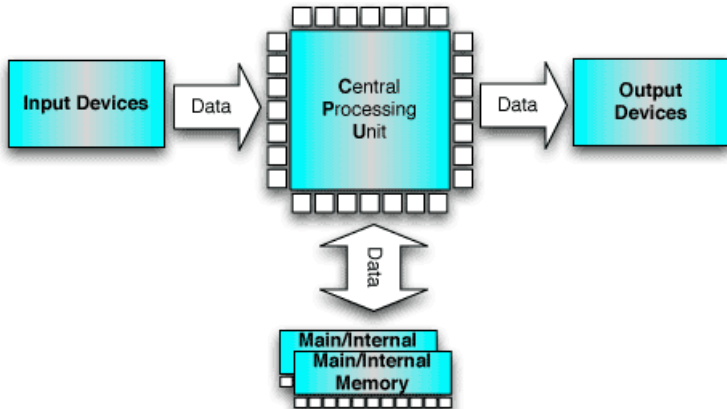
# Data Centers: Processing/Storing Huge volume of data



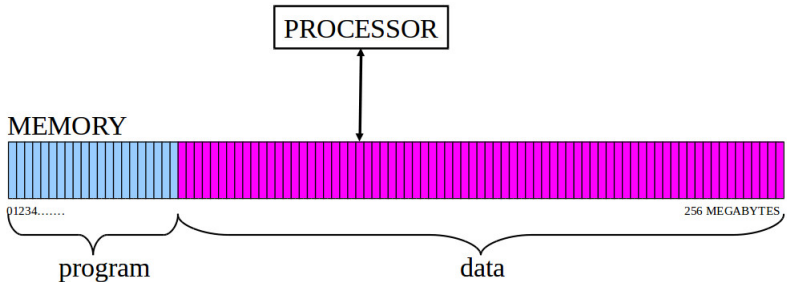
Even Cooling them is a big deal ...



# The Computing Machine : von Neuman Architecture



# The Computing Machine



- A program is a sequence of instructions assembled for some given task.
- Most instructions operate on data.
- Some instructions control the flow of the operations.

## How does the computer represent data?

- To store : Numbers, text, graphics and images, video, audio, program instructions.

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# How does the computer represent data?

- To store : Numbers, text, graphics and images, video, audio, program instructions.
- In some way, all information is digitized - broken down into pieces and represented as numbers.
- Example : Representing Text Digitally.
  - Every character is stored as a number, including spaces, digits, and punctuation.
  - Corresponding upper and lower case letters are separate characters.



# The ASCII table

American Standard Code for Information Interchange (ASCII).

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| Dec | Hx | Oct | Char                               | Dec | Hx | Oct | Html  | Chr   | Dec | Hx | Oct | Html  | Chr | Dec | Hx | Oct | Html   | Chr |
|-----|----|-----|------------------------------------|-----|----|-----|-------|-------|-----|----|-----|-------|-----|-----|----|-----|--------|-----|
| 0   | 0  | 000 | <b>NUL</b> (null)                  | 32  | 20 | 040 | &#32; | Space | 64  | 40 | 100 | &#64; | @   | 96  | 60 | 140 | &#96;  | `   |
| 1   | 1  | 001 | <b>SOH</b> (start of heading)      | 33  | 21 | 041 | &#33; | !     | 65  | 41 | 101 | &#65; | A   | 97  | 61 | 141 | &#97;  | a   |
| 2   | 2  | 002 | <b>STX</b> (start of text)         | 34  | 22 | 042 | &#34; | "     | 66  | 42 | 102 | &#66; | B   | 98  | 62 | 142 | &#98;  | b   |
| 3   | 3  | 003 | <b>ETX</b> (end of text)           | 35  | 23 | 043 | &#35; | #     | 67  | 43 | 103 | &#67; | C   | 99  | 63 | 143 | &#99;  | c   |
| 4   | 4  | 004 | <b>EOT</b> (end of transmission)   | 36  | 24 | 044 | &#36; | \$    | 68  | 44 | 104 | &#68; | D   | 100 | 64 | 144 | &#100; | d   |
| 5   | 5  | 005 | <b>ENQ</b> (enquiry)               | 37  | 25 | 045 | &#37; | %     | 69  | 45 | 105 | &#69; | E   | 101 | 65 | 145 | &#101; | e   |
| 6   | 6  | 006 | <b>ACK</b> (acknowledge)           | 38  | 26 | 046 | &#38; | &     | 70  | 46 | 106 | &#70; | F   | 102 | 66 | 146 | &#102; | f   |
| 7   | 7  | 007 | <b>BEL</b> (bell)                  | 39  | 27 | 047 | &#39; | '     | 71  | 47 | 107 | &#71; | G   | 103 | 67 | 147 | &#103; | g   |
| 8   | 8  | 010 | <b>BS</b> (backspace)              | 40  | 28 | 050 | &#40; | (     | 72  | 48 | 110 | &#72; | H   | 104 | 68 | 150 | &#104; | h   |
| 9   | 9  | 011 | <b>TAB</b> (horizontal tab)        | 41  | 29 | 051 | &#41; | )     | 73  | 49 | 111 | &#73; | I   | 105 | 69 | 151 | &#105; | i   |
| 10  | A  | 012 | <b>LF</b> (NL line feed, new line) | 42  | 2A | 052 | &#42; | *     | 74  | 4A | 112 | &#74; | J   | 106 | 6A | 152 | &#106; | j   |
| 11  | B  | 013 | <b>VT</b> (vertical tab)           | 43  | 2B | 053 | &#43; | +     | 75  | 4B | 113 | &#75; | K   | 107 | 6B | 153 | &#107; | k   |
| 12  | C  | 014 | <b>FF</b> (NP form feed, new page) | 44  | 2C | 054 | &#44; | ,     | 76  | 4C | 114 | &#76; | L   | 108 | 6C | 154 | &#108; | l   |
| 13  | D  | 015 | <b>CR</b> (carriage return)        | 45  | 2D | 055 | &#45; | -     | 77  | 4D | 115 | &#77; | M   | 109 | 6D | 155 | &#109; | m   |
| 14  | E  | 016 | <b>SO</b> (shift out)              | 46  | 2E | 056 | &#46; | .     | 78  | 4E | 116 | &#78; | N   | 110 | 6E | 156 | &#110; | n   |
| 15  | F  | 017 | <b>SI</b> (shift in)               | 47  | 2F | 057 | &#47; | /     | 79  | 4F | 117 | &#79; | O   | 111 | 6F | 157 | &#111; | o   |
| 16  | 10 | 020 | <b>DLE</b> (data link escape)      | 48  | 30 | 060 | &#48; | 0     | 80  | 50 | 120 | &#80; | P   | 112 | 70 | 160 | &#112; | p   |
| 17  | 11 | 021 | <b>DC1</b> (device control 1)      | 49  | 31 | 061 | &#49; | 1     | 81  | 51 | 121 | &#81; | Q   | 113 | 71 | 161 | &#113; | q   |
| 18  | 12 | 022 | <b>DC2</b> (device control 2)      | 50  | 32 | 062 | &#50; | 2     | 82  | 52 | 122 | &#82; | R   | 114 | 72 | 162 | &#114; | r   |
| 19  | 13 | 023 | <b>DC3</b> (device control 3)      | 51  | 33 | 063 | &#51; | 3     | 83  | 53 | 123 | &#83; | S   | 115 | 73 | 163 | &#115; | s   |
| 20  | 14 | 024 | <b>DC4</b> (device control 4)      | 52  | 34 | 064 | &#52; | 4     | 84  | 54 | 124 | &#84; | T   | 116 | 74 | 164 | &#116; | t   |
| 21  | 15 | 025 | <b>NAK</b> (negative acknowledge)  | 53  | 35 | 065 | &#53; | 5     | 85  | 55 | 125 | &#85; | U   | 117 | 75 | 165 | &#117; | u   |
| 22  | 16 | 026 | <b>SYN</b> (synchronous idle)      | 54  | 36 | 066 | &#54; | 6     | 86  | 56 | 126 | &#86; | V   | 118 | 76 | 166 | &#118; | v   |
| 23  | 17 | 027 | <b>ETB</b> (end of trans. block)   | 55  | 37 | 067 | &#55; | 7     | 87  | 57 | 127 | &#87; | W   | 119 | 77 | 167 | &#119; | w   |
| 24  | 18 | 030 | <b>CAN</b> (cancel)                | 56  | 38 | 070 | &#56; | 8     | 88  | 58 | 130 | &#88; | X   | 120 | 78 | 170 | &#120; | x   |
| 25  | 19 | 031 | <b>EM</b> (end of medium)          | 57  | 39 | 071 | &#57; | 9     | 89  | 59 | 131 | &#89; | Y   | 121 | 79 | 171 | &#121; | y   |
| 26  | 1A | 032 | <b>SUB</b> (substitute)            | 58  | 3A | 072 | &#58; | :     | 90  | 5A | 132 | &#90; | Z   | 122 | 7A | 172 | &#122; | z   |
| 27  | 1B | 033 | <b>ESC</b> (escape)                | 59  | 3B | 073 | &#59; | ;     | 91  | 5B | 133 | &#91; | [   | 123 | 7B | 173 | &#123; | {   |
| 28  | 1C | 034 | <b>FS</b> (file separator)         | 60  | 3C | 074 | &#60; | <     | 92  | 5C | 134 | &#92; | \   | 124 | 7C | 174 | &#124; |     |
| 29  | 1D | 035 | <b>GS</b> (group separator)        | 61  | 3D | 075 | &#61; | =     | 93  | 5D | 135 | &#93; | ]   | 125 | 7D | 175 | &#125; | }   |
| 30  | 1E | 036 | <b>RS</b> (record separator)       | 62  | 3E | 076 | &#62; | >     | 94  | 5E | 136 | &#94; | ^   | 126 | 7E | 176 | &#126; | ~   |
| 31  | 1F | 037 | <b>US</b> (unit separator)         | 63  | 3F | 077 | &#63; | ?     | 95  | 5F | 137 | &#95; | _   | 127 | 7F | 177 | &#127; | DEL |

## But, how does number get stored?

### Number Systems.

- Decimal (base 10 - uses 10 symbols  $\{0 \dots 9\}$ ). Eg : 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 ....

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## Quick Primer on Number System : Base $n$

Take every "digit" and multiply by increasing powers of  $n$  and add.

$$\begin{array}{ccc} & 329 & \\ / & | & \backslash \\ 10^2 & 10^1 & 10^0 \end{array}$$

$$3 \times 100 + 2 \times 10 + 9 \times 1 = 329$$

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$$3 \times 100 + 2 \times 10 + 9 \times 1 = 329$$

$$\begin{array}{ccc} & \text{most} & \text{least} \\ & \text{significant} & \text{significant} \\ & \leftarrow & \leftarrow \\ & 101 & \\ / & | & \backslash \\ 2^2 & 2^1 & 2^0 \end{array}$$

$$1 \times 4 + 0 \times 2 + 1 \times 1 = 5$$

## Converting from Decimal to Binary

Convert the decimal number 39 to binary (base 2).

|   |  |    |               |
|---|--|----|---------------|
| 2 |  | 39 |               |
| 2 |  | 19 | + Remainder 1 |
| 2 |  | 9  | + Remainder 1 |
| 2 |  | 4  | + Remainder 1 |
| 2 |  | 2  | + Remainder 0 |
| 2 |  | 1  | + Remainder 0 |
|   |  | 0  | + Remainder 1 |

$$\begin{aligned}39 &= 2 * 19 + 1 \\ &= 2 * (2 * 9 + 1) + 1 \\ &= 2^2 * 9 + 2^1 * 1 + 1 \\ &= 2^2 * (2 * 4 + 1) + 2^1 * 1 + 1 \\ &= 2^3 * 4 + 2^2 * 1 + 2^1 * 1 + 1 \\ &= 2^3 * (2 * 2 + 0) + 2^2 * 1 + 2^1 * 1 + 1 \\ &= 2^4 * 2 + 2^3 * 0 + 2^2 * 1 + 2^1 * 1 + 1 \\ &= 2^4 * (2 * 1 + 0) + \dots \\ &= 2^5 * 1 + 2^4 * 0 + 2^3 * 0 + 2^2 * 1 + 2^1 * 1 + 1\end{aligned}$$

$$\begin{aligned}(100111)_2 &= (1 \times 2^0) + (1 \times 2^1) + (1 \times 2^2) + (0 \times 2^3) + (0 \times 2^4) + (1 \times 2^5) \\ &= (39)_{10}\end{aligned}$$

# Which Number System? Binary !

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- The other number systems can be "encoded in" binary

Octal - 2 7 3 2 5 6 0 6

Binary - 010111011010101110000110

Hexadecimal - 5 13 10 11 8 6

5 D A B 8 6

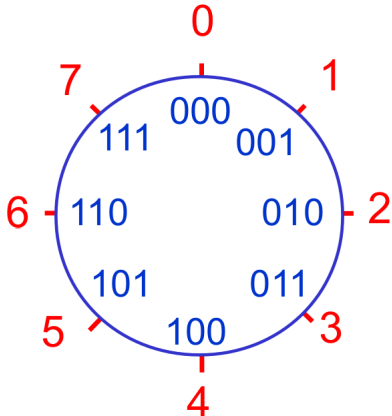


## Representing values in Binary

If we have  $m$  bits, we can represent  $2^m$  unique different values.

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A useful circle :



# Representing negative numbers

## Sign Magnitude notation

- Use one bit for sign, others for magnitude of the number.

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- Use one bit for sign, others for magnitude of the number.

|       | Sign Magn. |
|-------|------------|
| 0 0 0 | 0          |
| 0 0 1 | +1         |
| 0 1 0 | +2         |
| 0 1 1 | +3         |
| 1 0 0 | 0          |
| 1 0 1 | -1         |
| 1 1 0 | -2         |
| 1 1 1 | -3         |

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|       | Sign Magn. |
|-------|------------|
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| 0 1 1 | +3         |
| 1 0 0 | 0          |
| 1 0 1 | -1         |
| 1 1 0 | -2         |
| 1 1 1 | -3         |

- using  $n$  bits:  $-(2^{n-1} - 1) \dots (2^{n-1} - 1)$ .
- zero has two representations.

# Representing negative numbers

## Ones complement notation

- for a negative number  $n$ , represent the number by the bit complement of its binary rep. using  $k$  bits.

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|-------|------------|------------|
| 0 0 0 | 0          | 0          |
| 0 0 1 | +1         | +1         |
| 0 1 0 | +2         | +2         |
| 0 1 1 | +3         | +3         |
| 1 0 0 | 0          | -3         |
| 1 0 1 | -1         | -2         |
| 1 1 0 | -2         | -1         |
| 1 1 1 | -3         | 0          |

# Representing negative numbers

## Ones complement notation

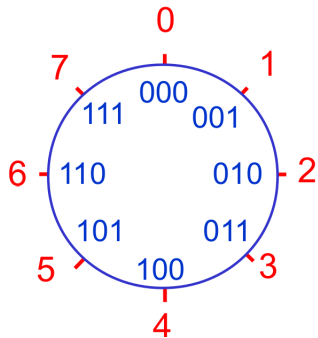
- for a negative number  $n$ , represent the number by the bit complement of its binary rep. using  $k$  bits.

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| 1 0 0 | 0          | -3         |
| 1 0 1 | -1         | -2         |
| 1 1 0 | -2         | -1         |
| 1 1 1 | -3         | 0          |

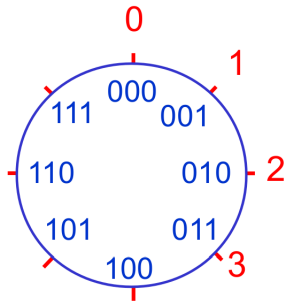
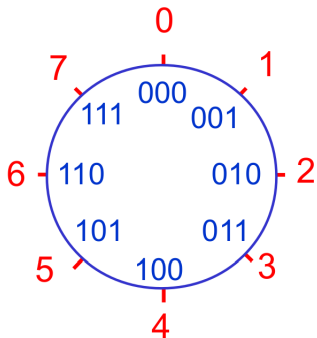
- using  $n$  bits:  $-(2^{n-1} - 1) \dots (2^{n-1} - 1)$ .
- zero has two representations.
- not very widely used representation.



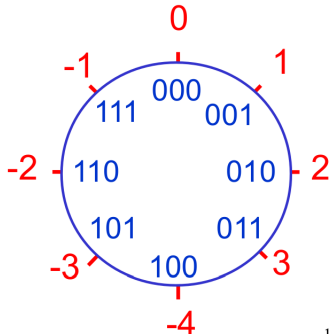
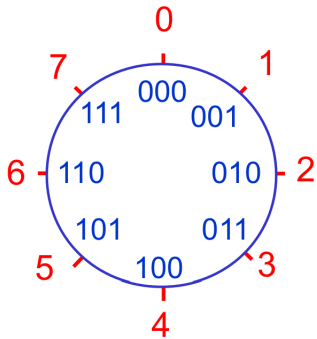
## Representing negative numbers - A neat trick



## Representing negative numbers - A neat trick



## Representing negative numbers - A neat trick



# Representing negative numbers - A neat trick

## Twos complement notation

- for a positive number  $n$ , represent the number by its binary rep. using  $k$  bits.
- for a negative number  $-n$ , represent the number as  $2^k - n$ .

## Representing negative numbers - A neat trick

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|       | Sign Magn. | Ones comp. | Twos comp. |
|-------|------------|------------|------------|
| 0 0 0 | 0          | 0          | 0          |
| 0 0 1 | +1         | +1         | +1         |
| 0 1 0 | +2         | +2         | +2         |
| 0 1 1 | +3         | +3         | +3         |
| 1 0 0 | 0          | -3         | -4         |
| 1 0 1 | -1         | -2         | -3         |
| 1 1 0 | -2         | -1         | -2         |
| 1 1 1 | -3         | 0          | -1         |

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| 1 0 1 | -1         | -2         | -3         |
| 1 1 0 | -2         | -1         | -2         |
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- using  $n$  bits:  $-(2^{n-1}) \dots (2^{n-1} - 1)$ .
- widely used representation.

## Representing negative numbers

Arithmetic with these representations

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| 0 0 1 | +1         | +1         | +1         |
| 0 1 0 | +2         | +2         | +2         |
| 0 1 1 | +3         | +3         | +3         |
| 1 0 0 | 0          | -3         | -4         |
| 1 0 1 | -1         | -2         | -3         |
| 1 1 0 | -2         | -1         | -2         |
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| 1 0 0 | 0          | -3         | -4         |
| 1 0 1 | -1         | -2         | -3         |
| 1 1 0 | -2         | -1         | -2         |
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- $2 + (-3)$



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| 0 1 0 | +2         | +2         | +2         |
| 0 1 1 | +3         | +3         | +3         |
| 1 0 0 | 0          | -3         | -4         |
| 1 0 1 | -1         | -2         | -3         |
| 1 1 0 | -2         | -1         | -2         |
| 1 1 1 | -3         | 0          | -1         |

- $2 + (-3)$
- $3 + (-2)$

## More examples : The case of 4 bits

|                |            |
|----------------|------------|
| 1 1            | corresp.   |
| √√√√           | dec. oper. |
| 0110           | +6         |
| +1101          | + -3       |
| <hr/>          |            |
| 10011 = +3     | +3         |
| └──┘           |            |
| correct result |            |


Example (a)



|                |            |
|----------------|------------|
|                | corresp.   |
|                | dec. oper. |
| 0100           | +4         |
| +1001          | + -7       |
| <hr/>          |            |
| 1101 = -3      | -3         |
| └──┘           |            |
| correct result |            |

correct result

Example (b)

## More examples : The case of 4 bits

|   | corresp.<br>dec. oper. |
|---|------------------------|
| 0011  | +3                     |
| +0100   | + +4                   |
| <hr/>   |                        |
| 0111 = +7   | +7                     |
|  |                        |
| correct result  |                        |
| Example (c)   |                        |

|   | corresp.<br>dec. oper. |
|---|------------------------|
| 1 1 1   |                        |
|  |                        |
| 1110  | -2                     |
| +1010   | + -6                   |
| <hr/>   |                        |
| 11000 = -8  | -8                     |
|  |                        |
| correct result  |                        |
| Example (d)   |                        |

## More examples : The case of 4 bits

| 1<br>√     | corresp.<br>dec. oper. |
|------------|------------------------|
| 1101       | -3                     |
| +1010      | + -6                   |
| <hr/>      |                        |
| 10111 = +7 | -9                     |

incorrect result

Example (e)

| 1<br>√    | corresp.<br>dec. oper. |
|-----------|------------------------|
| 0101      | +5                     |
| +0110     | + +6                   |
| <hr/>     |                        |
| 1011 = -5 | +11                    |

incorrect result

Example (f)

**Overflow Detection Rule** : If two numbers with the same sign (both positive or both negative) are added, then overflow occurs if and only if the binary representation of the result has the opposite sign.

# What to do?

**How to Detect it?** : The technique of overflow detection is easily implemented in electronic circuitry, and it is a standard feature in digital adder circuits.

**How to Prevent it?:** Use more bits!