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Fun fact: sometimes Task 1 can be done using Task 2.

(Computing $f(x) = \sqrt{x}$ at $x = k$) **is same as** (Root finding for $f(x) = x^2 - k$).

Find $f(x)$ given x

f be a function in one variable. f' , f'' denote the derivatives of increasing order.

The McLaurin series for f :

$$\begin{aligned} f(x) = & f(0) + f'(0)x \\ & + f''(0)\frac{x^2}{2!} + \dots \end{aligned}$$

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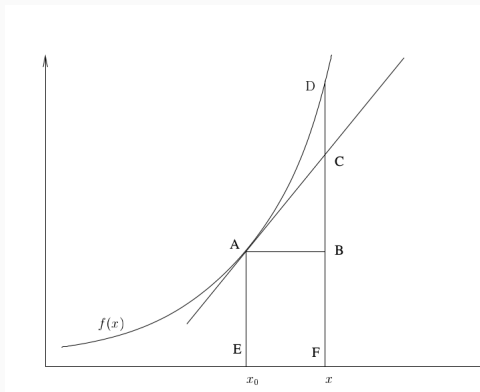
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In general, the Taylor series expansion for the function is :

$$\begin{aligned} f(x) = & f(x_0) + f'(x_0)(x - x_0) \\ & + f''(x_0)\frac{(x - x_0)^2}{2!} + \\ & \dots \end{aligned}$$



$$f(x) \approx f(x_0) + f'(x_0) \cdot (x - x_0)$$

Example : Computing the Sine of an Angle

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C-program segment

```
double x;
scanf("%lf",&x);

double err = 1.0E-10, sum = x;
double term = x;

for (int k=2; abs(term) > err; k++)
{
    term *= -x * x / ((2*k-2)*(2*k-1));
    sum += term;
}

printf("sin(%lf) = %lf",x,sum);
```

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Idea : The root of a function is a value x_0 where the plot of the function touches the x -axis.

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Bisection Method: The method requires that we are given two values of x , x_L and x_R such that $f(x_L)$ and $f(x_R)$ have opposite signs (there is a root for $f(x)$ lying between them).

- Find $x_M = \frac{x_L + x_R}{2}$. Find out $f(x_M)$.

If sign of $f(x_M)$ and $f(x_L)$ are the same, assign $x_L = x_M$.

If sign of $f(x_M)$ and $f(x_R)$ are the same, assign $x_R = x_M$.

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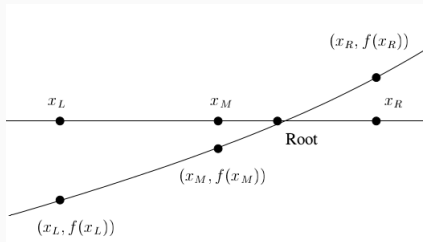
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Guaranteeing a zero in that interval.

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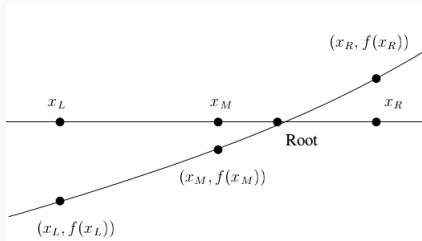


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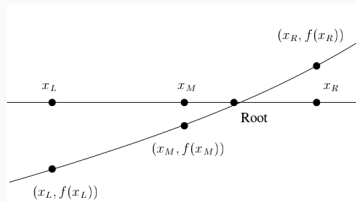
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For the above example,
assign $x_L = x_M$ and continue.

Bisection Method : Reading the Figure

- x_L and x_R are chosen so that $f(x_L)$ and $f(x_R)$ have opposite signs. Hence, for a continuous function, a root lies between them.
- $x_M = \frac{x_L + x_R}{2}$ is the midpoint of the current interval.
- In the figure, $f(x_L) < 0$, $f(x_M) < 0$, and $f(x_R) > 0$. Therefore, x_L and x_M are on the same side of the x -axis.
- We discard the interval $[x_L, x_M]$, set $x_L = x_M$, and continue with $[x_M, x_R]$.
- Each iteration halves the interval while keeping a root inside it.



Bisection Method : Program Segment

```
// Assume #include<stdio.h>,#include<stdbool.h>, main() are written

float xL=0,xR=2,xM; // invariant: f(xL),f(xR) have different signs.

bool xLIsPositive = (xL*xL - 2) > 0;
// Invariant: xLIsPositive gives the sign of f(x_L).
bool xMIsPositive;

float epsilon;
scanf ("%f", &epsilon);

while(xR-xL >= epsilon){
    xM = (xL+xR)/2;
    xMIsPositive = (xM*xM -2) > 0;
    if (xLIsPositive == xMIsPositive)
        xL = xM;
    else
        xR = xM;
}
```