CS3300 - Compiler Design

Liveness analysis and Register allocation

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Register allocation

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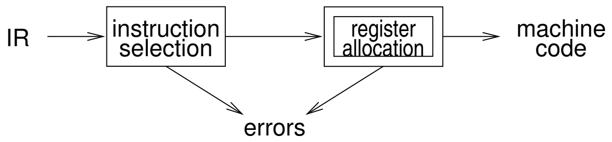
Register allocation



Register allocation:

- have value in a register when used
- limited resources
- can effect the instruction choices
- can move loads and stores
- optimal allocation is difficult
  $\Rightarrow$ NP-complete for $k \geq 1$ registers

Liveness analysis

Problem:

- IR contains an unbounded number of temporaries
- machine has bounded number of registers

Approach:

- temporaries with disjoint live ranges can map to same register
- if not enough registers then spill some temporaries
  (i.e., keep them in memory)

The compiler must perform liveness analysis for each temporary:

It is live if it holds a value that may be needed in future

Example

$$
\begin{array}{ll}
 & a \leftarrow 0 \\
L_1: & b \leftarrow a+1 \\
 & c \leftarrow c+b \\
 & a \leftarrow b \times 2 \\
 & \text{if } a < N \text{ goto } L_1 \\
 & \text{return } c
\end{array}
$$

Liveness analysis

Gathering liveness information is a form of data flow analysis operating over the CFG:

- We will treat each statement as a different basic block.
- liveness of variables "flows" around the edges of the graph
- assignments define a variable, v:
  - $def(v)$ = set of graph nodes that define v
  - $def[n]$ = set of variables defined by n
- occurrences of v in expressions use it:
  - $use(v)$ = set of nodes that use v
  - $use[n]$ = set of variables used in n

Definitions

- v is live on edge e if there is a directed path from e to a use of v that does not pass through any $def(v)$
- v is live-in at node n if live on any of n's in-edges
- v is live-out at n if live on any of n's out-edges
- $v \in use[n] \Rightarrow v$ live-in at n
- (For programs with statically established no uninitialized variables) v live-in at $n \Rightarrow v$ live-out at all $m \in pred[n]$
- v live-out at $n, v \notin def[n] \Rightarrow v$ live-in at n

Liveness analysis

Define:

$$
\begin{aligned}
in[n] &= \text{variables live-in at } n \\
out[n] &= \text{variables live-out at } n
\end{aligned}
$$

Then:

$$
out[n] = \bigcup_{s \in succ(n)} in[s]
$$

$$
succ[n] = \phi \Rightarrow out[n] = \phi
$$

Note:

$$
\begin{aligned}
in[n] &\supseteq use[n] \\
in[n] &\supseteq out[n] - def[n]
\end{aligned}
$$

$use[n]$ and $def[n]$ are constant (independent of control flow)
Now, $v \in in[n]$ iff. $v \in use[n]$ or $v \in out[n] - def[n]$
Thus, $in[n] = use[n] \cup (out[n] - def[n])$

Iterative solution for liveness

N : Set of nodes of CFG;
foreach $n \in N$ **do**
  $in[n] \leftarrow \phi$;
  $out[n] \leftarrow \phi$;
end
repeat
  foreach n ∈ Nodes **do**
    $in'[n] \leftarrow in[n]$;
    $out'[n] \leftarrow out[n]$;
    $in[n] \leftarrow use[n] \cup (out[n] - def[n])$;
    $out[n] \leftarrow \bigcup_{s \in succ[n]} in[s]$;
  end
until $\forall n, in'[n] = in[n] \wedge out'[n] = out[n]$;

Notes

- should order computation of inner loop to follow the "flow"
- liveness flows backward along control-flow arcs, from out to in
- nodes can just as easily be basic blocks to reduce CFG size
- could do one variable at a time, from uses back to defs, noting liveness along the way

Iterative solution for liveness

Complexity: for input program of size N

- $\leq N$ nodes in CFG
  $\Rightarrow \leq N$ variables
  $\Rightarrow N$ elements per *in/out*
  $\Rightarrow O(N)$ time per set-union
- **for** loop performs constant number of set operations per node
  $\Rightarrow O(N^2)$ time for **for** loop
- each iteration of **repeat** loop can only add to each set
  sets can contain at most every variable
  $\Rightarrow$ sizes of all in and out sets sum to $2N^2$,
  bounding the number of iterations of the **repeat** loop

$\Rightarrow$ worst-case complexity of $O(N^4)$

- ordering can cut **repeat** loop down to 2-3 iterations
  $\Rightarrow O(N)$ or $O(N^2)$ in practice

Least fixed points

There is often more than one solution for a given dataflow problem (see example).
Any solution to dataflow equations is a conservative approximation:

- v has some later use downstream from n
  $\Rightarrow v \in out(n)$
- but not the converse

Conservatively assuming a variable is live does not break the program; just means more registers may be needed.
Assuming a variable is dead when really live will break things.
Many possible solutions but we want the "smallest": the least fixpoint.
The iterative algorithm computes this least fixpoint.

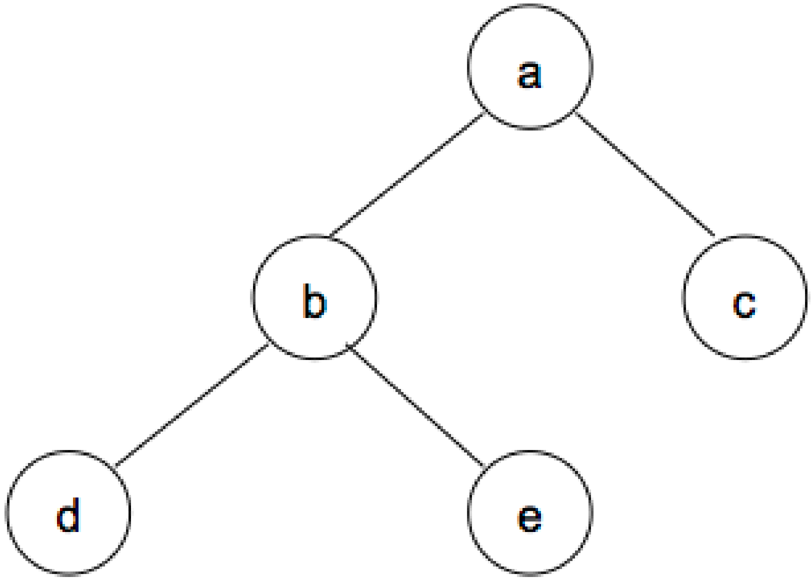
Register allocation - by Graph coloring

- Step 1:
  - Select target machine instructions assuming infinite registers (temps).
  - If a instruction requires a special register – replace that temp with that register.
- Step 2:
  - Construct an interference graph.
  - Solve the register allocation problem by coloring the graph.
  - A graph is said to be colored if each each pair of neighboring nodes have different colors.

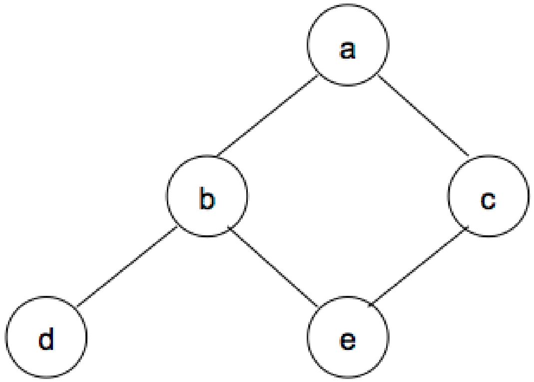
Graph coloring - a simplistic approach

Input: G - the interference graph, K - number of colors
repeat
    repeat
        Remove a node n and all its edges from G, such that degree of n is less than K;
        Push n onto a stack;
    until G has no node with degree less than K;
    `//` G `is either empty or all of its nodes have degree` $\geq$ `K`
    if G is not empty **then**
        Take one node m out of G, and mark it for spilling;
        Remove all the edges of m from G;
    end
until G is empty;
Take one node at a time from the stack and assign a non conflicting color.

Example 1, available colors = 2



Example 2



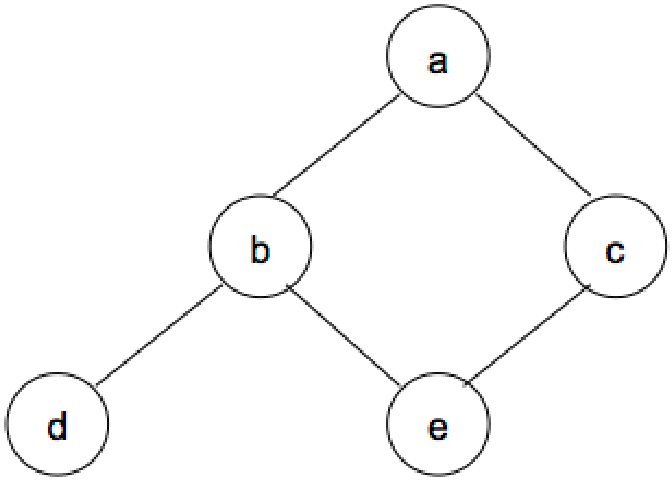
We have to spill.

Graph coloring - Kempe's heuristic

- Algorithm dating back to 1879.

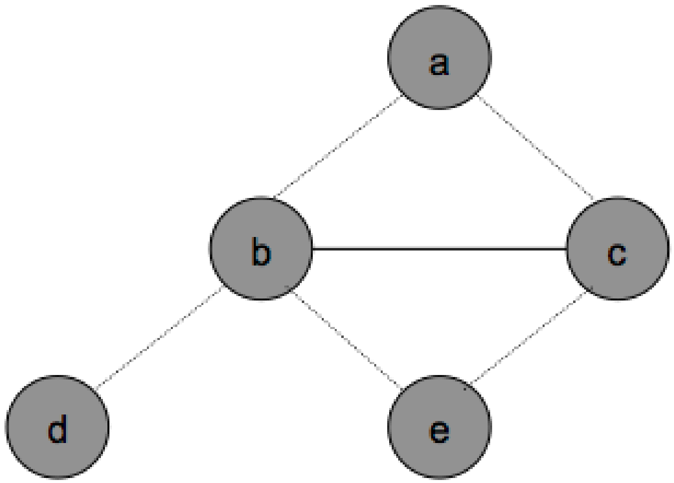
Input: G - the interference graph, K - number of colors
repeat
  repeat
    Remove a node n and all its edges from G, such that degree of n is less than K;
    Push n onto a stack;
  until G has no node with degree less than K;
  // G is either empty or all of its nodes have degree $\geq$ K
  if G is not empty **then**
    Take one node m out of G.;
    push m onto the stack;
  end
until G is empty;
Take one node at a time from the stack and assign a non conflicting color (if possible, else spill).

Example 2 (revisited)



We don't have to spill.

Example 3



Don't have a choice. Have to spill.

Register allocation - Linear scan

Register allocation is **expensive**.

- Many algorithms use heuristics for graph coloring.
- Allocation may take time quadratic in the number of live intervals.

Not suitable

- Online compilers – need to generate code quickly. e.g. JIT compilers.
- Sacrifice efficient register allocation for compilation speed.

Linear scan register allocation - Massimiliano Poletto and Vivek Sarkar, ACM TOPLAS 1999

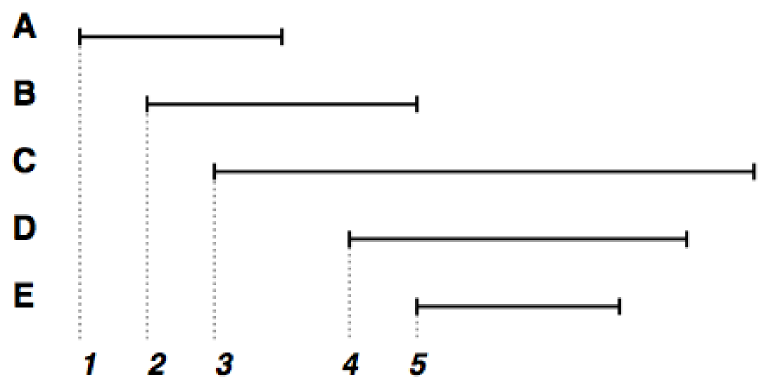
Linear Scan algorithm

```
LINEARSCANREGISTERALLOCATION
    active ← {}
    foreach live interval i, in order of increasing start point
        EXPIREOLDINTERVALS(i)
        if length(active) = R then
            SPILLATINTERVAL(i)
        else
            register[i] ← a register removed from pool of free registers
            add i to active, sorted by increasing end point

EXPIREOLDINTERVALS(i)
    foreach interval j in active, in order of increasing end point
        if endpoint[j] ≥ startpoint[i] then
            return
        remove j from active
        add register[j] to pool of free registers

SPILLATINTERVAL(i)
    spill ← last interval in active
    if endpoint[spill] > endpoint[i] then
        register[i] ← register[spill]
        location[spill] ← new stack location
        remove spill from active
        add i to active, sorted by increasing end point
    else
        location[i] ← new stack location
```


Example



- Say, available registers = 2

Linear Scan algorithm - analysis

- Each live range gets either a register or a spill location.
- Note: The number of overlapping intervals changes only at the start and end points of an interval.
- Live intervals are stored in a list that is sorted in order of increasing start point.
- The active list is kept sorted in order of increasing end point. Adv: need to scan only those intervals (+1 at most) that have to be removed.
- Complexity: $O(V)$ – if number of registers is assumed ot be a constant. Else? $O(V \times logR)$

Spilling

- We need to generate extra instructions to load variables from the stack and store them back.
- The load and store may require registers again:
  - Naive approach: Keep a separate register (wasteful).
  - Rewrite the code - by introducing a temporary; rerun the liveness + ra.
    (Note: the new temp has much smaller live range).

Example: rewrite code

Consider: `add t1 t2`

- Suppose `t2` has to be spilled, say to `[sp-4]`.
- Invent a new temp `t35`, and rewrite:

```
mov t35 [sp-4]
add t1 t35
```

- `t35` has a very short live range and less likely to interfere.
- Now rerun the algo.

Criteria for spilling

During register allocation, we identify that one of the live ranges from a given set, has to be spilled. Criteria?

- Random! Adv? Disadv?
- One with maximum degree
- One that has the longest life
- One with the shortest life (take advantage of the cache).
- One with least cost.
  - Cost = Dynamic (load cost + store cost)
  - How to handle loops, conditionals?
  - Cost of load, store