Lists

Rupesh Nasre.
rupesh@cse.iitm.ac.in

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ADT

- Abstract Data Type
- Defines the interface of the functionality provided by the data structure.
- Hides implementation details.
  - Defines *what* and hides *how*.
- Makes software modular.
- Allows easy change of implementation.

List as an ADT

```
class List {
public:
  List();
  void insert(Element e);
  void find(Element e);
  void remove(Element e);
  void print();
  int size();
};
```

What are the complexities of these operations?

Other ADTs

- Fan regulator
  - IncSpeed, decSpeed, getSpeed, getCompanyName
- Integer
  - size, isSigned, getValue, setValue, add, sub
- Student
  - getRollNo, getHostel, getFavGame, setHostel, getSlots, setCGPA

List using Array

```
class List {
public:
  List();
  void insert(Element e);
  void find(Element e);
  void remove(Element e);
  void print();
  int size();
};
```

4	2	7	2	9			

Design decisions

- Size of the array?
- Maintain size separately or use a sentinel?
- On overflow: error or realloc?
- On underflow: error message or exit or silent?
- Printing order?
- Duplicates allowed?
- For duplicates, what does remove do?
- ...

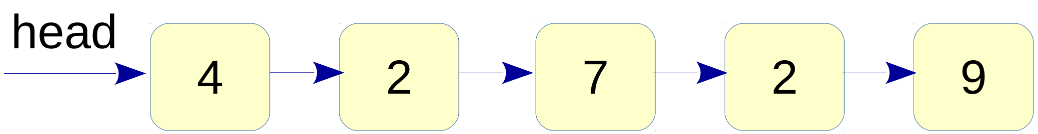
List using Array

4	2	7	2	9			

With certain design decisions:

```
class List {
public:
  List();
  void insert(Element e);   O(1)
  void find(Element e);     O(N)
  void remove(Element e);   O(N)
  void print();             O(N)
  int size();               O(1)
};
```

List using Linked List




```
class List {
public:
  List();
  void insert(Element e);
  void find(Element e);
  void remove(Element e);
  void print();
  int size();
};
```

- `void insert(Element e);` — **O(N) without tail pointer, else O(1)**
- `void find(Element e);` — **O(N)**
- `void remove(Element e);` — **O(N)**
- `void print();` — **O(N)**
- `int size();` — **O(1)**

If the complexities of array-based versus linked-list-based implementations are the same, **why use linked lists?**

Arrays versus Linked Lists

- Need to copy the existing array on reallocation.
- Removal of i^{th} element needs element-shifting from i+1 to end.
- Same with insertion.
- Array concatenation is linear time.

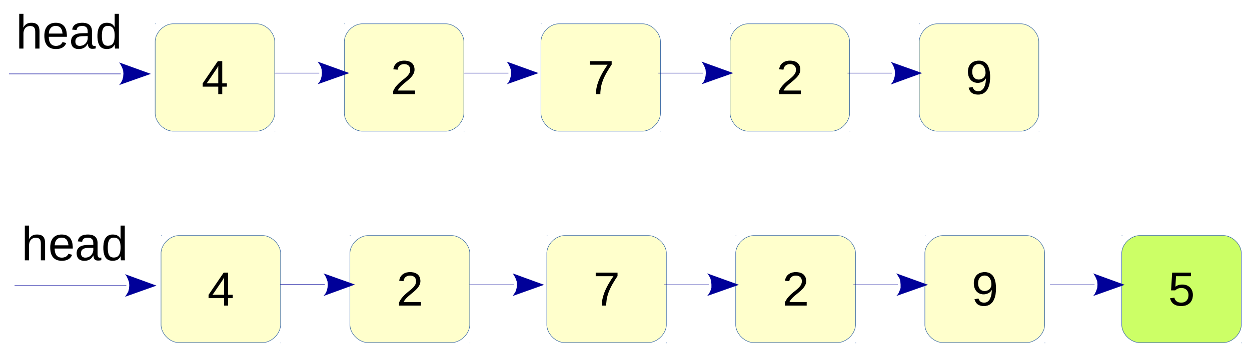
- Only a link needs to be established (O(1)).
- Removal of an element using pointers can be done in O(1).
- Same with insertion.
- List concatenation is O(1).

Linked List Implementation

- **Source: sll.cpp**

List insert

- insert(5)



Setup node:

```
  Node *newptr = new Node();
  newptr->val = 5;
  newptr->next = NULL;
```

End case:

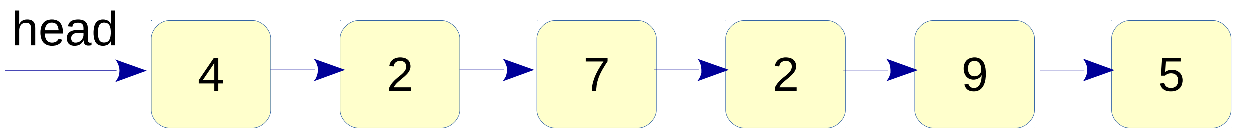
```
      if (head == NULL) head = newptr;
```

Regular case:

```
      for (Node *ptr = head; ptr->next; ptr = ptr->next)
            ;
      ptr->next = newptr;
```

List print

- print()



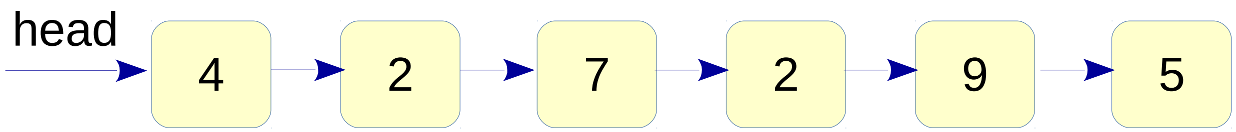
Output: 4 2 7 2 9 5

For each element in the list
    Print the element

```
for (Node *ptr = head; ptr; ptr = ptr->next)
      printf("%c ", ptr->val);
printf("\n");
```

List find

- find(9)



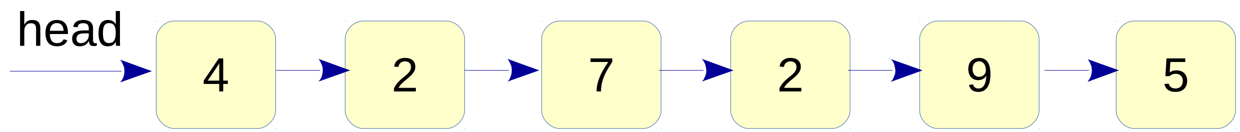
For each element in the list
    If the element is same as that to be searched
        Found the element
Element not present

```
for (Node *ptr = head; ptr; ptr = ptr->next)
    if (ptr->val == val) return true;
return false;
```

List remove

- remove(2)
- remove(5)
- remove(4)

We want to remove all occurrences of the value.



Special case:

```
if (head == NULL) return false;
```

General case:

```
Node *previous = NULL;
for (Node *ptr = head; ptr;) {
    if (ptr->val == val) {
        Node *toberemoved = ptr;
        if (previous) {
            previous->next = ptr->next;
        } else head = ptr->next;
        ptr = ptr->next;
        delete toberemoved;
        removed = true;
    } else {
        previous = ptr;
        ptr = ptr->next;
    }
}
```

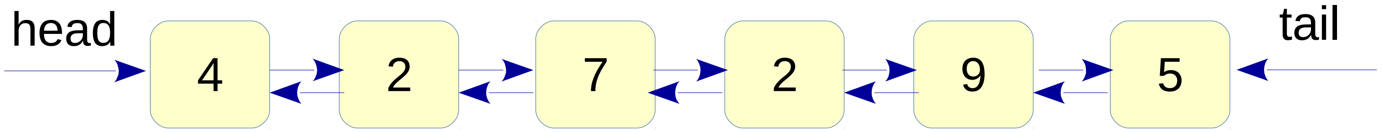
Pitfalls

- ptr = head->next; // segfault. Check if head is NULL.
- Node *ptr = &node1; return; // local variable node1.
- ptr = malloc(sizeof(Node*)); // insufficient memory.

Wrong **deleteList** program

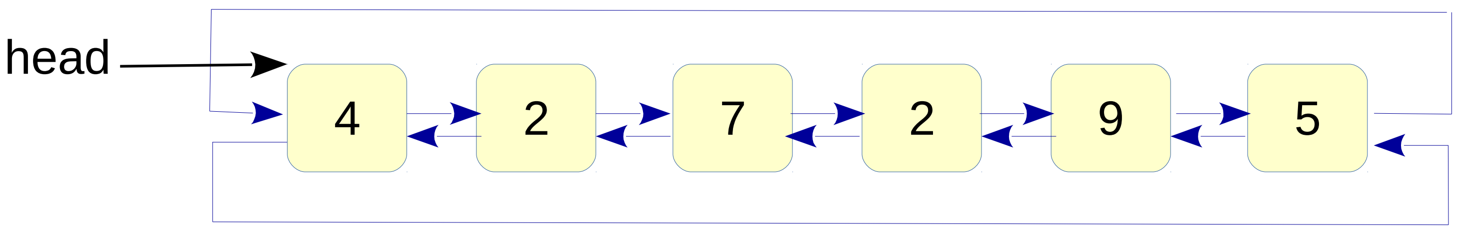
```
for (ptr = head; ptr; ptr = ptr->next)
    free(ptr);                              // invalid memory on free.
                                            // may work but wrong.
```

Doubly Linked List



- Links in both the directions.
- Node structure contains two pointers: next and previous.
- Deletion now becomes simpler.
- Two pointers: head and tail maintain list ends.
- **Classwork**: Write a function to remove a node.

Circular Doubly Linked List



- Last element points to the first, and first element's previous is the last node.
- Node structure continues to contain two pointers: next and previous.
- Tail pointer is not required.
- A singly linked list can also be circular.
- **Classwork**: Write a function to print all the node values in a CDLL.

Polynomial ADT

- $F(X) = \sum_{i=0}^{N} A_i X^i$
- Example: $x^4 - 4x^3 + 7x - 6$
- **Member functions**
  - Initialize
  - Set a coefficient (for a power)
  - Add polynomials
  - Multiply polynomials
  - ...

- **Implementation**
  - Could be using arrays
  - Could be using linked lists

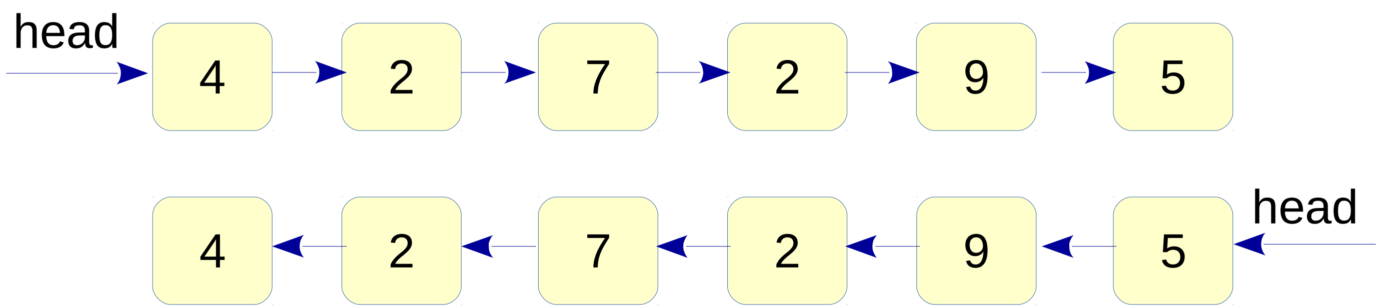
  - **Classwork:** Create a struct / class to implement polynomials.

- Are there disadvantages of using arrays?
  - $2x^{1000} - x$
  - What are the design decisions for using lists?

Polynomial ADT

```
class Polynomial {
    int coeff[MaxDegree + 1];
};
void Polynomial::initialize(int coeff[ ]) {
    // Classwork: implement this.
}
void Polynomial::add(Polynomial p2, Polynomial psum) {
    // Classwork: implement this.
}
```

List Reversal



- Given a list (SLL, DLL, CSLL, CDLL), reverse it.
- The traversal from head should result in the opposite order.
- Typically need three pointers: previous, current and next.
- **Classwork**: Write a list reversal for SLL (*sll.cpp*).
- **Classwork**: Write a recursive list reversal.

Recursive Methods

- Sometimes natural to model.
- Sometimes inefficient to implement.
- **Classwork**: find an element recursively.
- **Classwork**: print a list recursively.
  - How to print in reverse?
  - **sll.cpp**

Stack ADT

- Special List
- Operations restricted to one end.
- Insert --> Push
- Remove --> Pop
- LIFO
- Cannot access arbitrary element.
- **Important**: Since this is ADT, we do not care about the implementation yet.

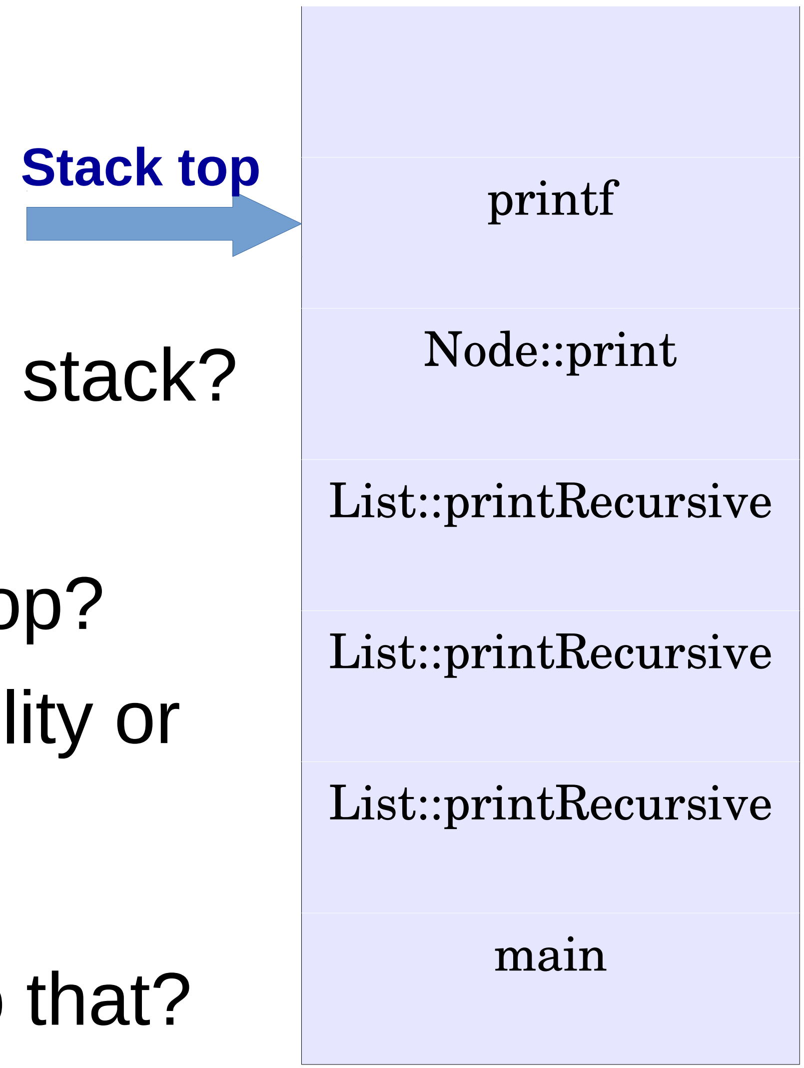
List versus Stack

class **List** {
  void insert(Element);
  void remove(Element);
  void search(Element);
  int size();
  void print();
  ...
};

class **Stack** {
  void push(Element);
  void pop(~~Element~~);
  ~~void search(Element)~~;
  ~~int size()~~; bool isEmpty();
  ~~void print();~~
  ...
};

Stack Implementation

- **Design decisions**
  - Array versus Linked List
  - Allow traversing through the stack?
  - Allow querying stack size?
  - Allow peeking at the stack top?
  - IsEmpty is user's responsibility or library implementation's?
  - Stack Top points to the last element, or the entry next to that?



Source: stack.cpp

Balanced Parentheses

- We want to check if parentheses are balanced or not.
- Three types of parentheses: ( ), [ ] and { }
- Valid inputs:
  - ([ ] [{ }])
  - [ ] { } [ ] ( ) [[[ ]]]
- Invalid inputs:
  - ((( ))
  - ([ )] { }
  - } }) ({ {

Classwork: Use stack to design an algorithm to check for balanced parentheses.

Question: Can we design an application of stack from its ADT without knowing its implementation?

Balanced Parentheses

```
for each input symbol c
  if (c is an open parenthesis) stack.push(c)
  else if (c is a close parenthesis) {
    if stack.top contains the matching open parenthesis
       pop the element from stack
    else error
  }
if (stack is empty)
    // all good.
else error
```

Find a string to match this error.

Source: parentheses.cpp

Find a string to match this error.

Stack Implementation

- stackimpl.c

Expressions

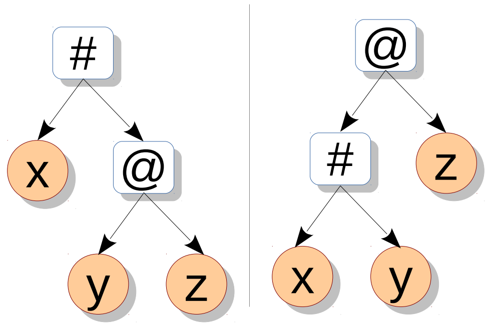
- 1 + 2 * 3 – 4
  - Binary operators appear between the operands
  - Ambiguous without extra knowledge

    (1 + 2) * (3 – 4) OR

    1 + (2 * (3 – 4)) OR

    (1 + (2 * 3)) – 4 OR

    ((1 + 2) * 3) – 4 ?



  - Parentheses help disambiguate; domain knowledge helps disambiguate (operator precedence).
  - Won’t it be nice if expressions can be written in unambiguous manner?

Prefix and Postfix Forms

- 1 + 2 * 3 – 4
  - Binary operators appear between the operands.
  - Called as **infix** form.
- 1 2 3 * + 4 -
  - Binary operators appear after the operands.
  - Called as **postfix** form.
- - + 1 * 2 3 4
  - Binary operators appear before the operands.
  - Called as **prefix** form.

How do these forms help resolve ambiguity?

Prefix, Postfix and Non-ambiguity

Infix	Prefix	Postfix
(1 + 2) * (3 – 4)		
1 + (2 * (3 – 4))		
(1 + (2 * 3)) – 4		
((1 + 2) * 3) – 4		
1 + ((2 * 3) - 4)		

Prefix, Postfix and Non-ambiguity

Infix	Prefix	Postfix
(1 + 2) * (3 – 4)	* + 1 2 – 3 4	1 2 + 3 4 - *
1 + (2 * (3 – 4))	+ 1 * 2 – 3 4	1 2 3 4 - * +
(1 + (2 * 3)) – 4	- + 1 * 2 3 4	1 2 3 * + 4 -
((1 + 2) * 3) – 4	- * + 1 2 3 4	1 2 + 3 * 4 -
1 + ((2 * 3) - 4)	+ 1 - * 2 3 4	1 2 3 * 4 - +

- No parentheses in prefix and postfix forms.
- Infix is ambiguous; prefix and postfix are not.
- Unique prefix and postfix forms for different orders of operator evaluation.

Postfix Evaluation

- Find the value of 5 1 2 3 * – 4 + 6 * –.
- Write a program to evaluate a postfix expression.
  - Assume digits, +, –, *, /.

```
For each symbol in the expression
    If the symbol is an operand
        Push its value to a stack
    Else if the symbol is an operator
        Pop two nodes from the stack
        Apply the operator on them
        Push result to the stack
```

Source: postfixeval.cpp

Prefix Evaluation

For each symbol in the expression right-to-left
    If the symbol is an **operand**
        Push its value to the stack
    Else if the symbol is an **operator**
        Pop two symbols from the stack
        Apply the operator on them
        Push result to the stack

Prefix
* + 1 2 – 3 4
+ 1 * 2 – 3 4
- + 1 * 2 3 4
- * + 1 2 3 4
+ 1 - * 2 3 4

Homework: Code this up.

Infix to Posfix

- Given an infix expression (with parentheses), convert it to a postfix form (without parentheses).

Infix	Prefix	Postfix
(1 + 2) * (3 – 4)	* + 1 2 – 3 4	1 2 + 3 4 - *
1 + (2 * (3 – 4))	+ 1 * 2 – 3 4	1 2 3 4 - * +
(1 + (2 * 3)) – 4	- + 1 * 2 3 4	1 2 3 * + 4 -
((1 + 2) * 3) – 4	- * + 1 2 3 4	1 2 + 3 * 4 -
1 + ((2 * 3) - 4)	+ 1 - * 2 3 4	1 2 3 * 4 - +

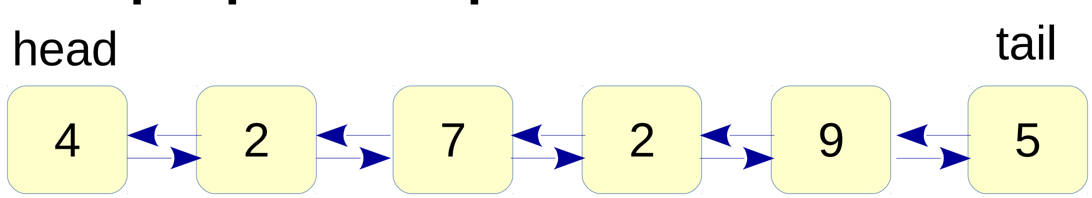
```
For each symbol in the expression
    If the symbol is an operand
        Print the symbol
    Else if the symbol is an opening parenthesis
        Push the symbol on stack
    Else if the symbol is a closing parenthesis
        Do {
            Pop symbol from the stack
            If symbol is not opening parenthesis
                Print the symbol
            } while symbol is not opening parenthesis
    Else {          // symbol c is an operator
            Pop symbol d from the stack
            While symbol d has higher or equal priority than c
                Print the symbol d
                Pop symbol d from the stack
            Push the symbol on stack
        }
    }
    While stack is not empty {
        Pop symbol from the stack
        Print the symbol
    }
    Return postfix
```


Source: Infix2postfix.cpp

Queue

- Special list
- Insertions at one end, deletions at the other
- Tracked using two pointers: head and tail
- FIFO (what is FCFS?)
- Cannot access arbitrary element
- Insert → push / enqueue

  remove → pop / dequeue



Queue ADT

- **Classwork**: Write down the Queue ADT.

```
struct Queue {
    void push(Element);      // enqueue
    Element pop();           // dequeue
    bool isEmpty();
    ...
};

class Queue {
    void push(Element);
    void pop();
    Element front();
    Element back();
    bool isEmpty();
    ...
};
```

Source: q.cpp

Call Center

- Multiple users call a call-center.
- Multiple operators answer the call.
- Each call takes an unknown amount of time.
- When all the operators are busy
  - Calling users need to wait.
- When an operator becomes available
  - Which waiting user is answered?
- Can we use Queue ADT to implement this?

Call Center: Data Structures

- User (id, call time)
- Operator (id)
- Queue of waiting users
- List of busy operators
- Queue of free operators

Call Center: Simulation

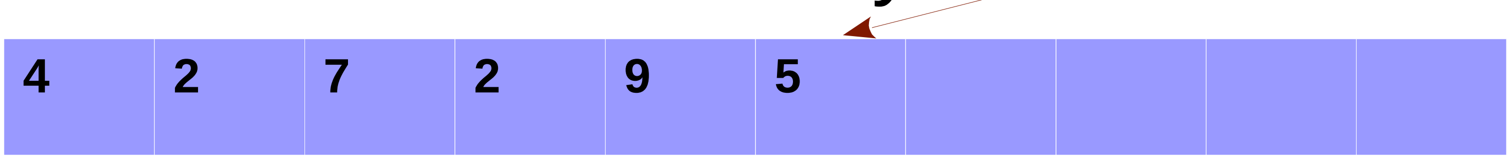
- Simulation is often based on time.
- At each time unit, various actions occur.
  - A new user arrives.
  - A free operator needs to be assigned to a user.
  - No operator is free, so the user needs to wait.
  - A busy operator becomes free.
  - Nothing happens, call time of engaged users reduces.
- Simulation ties these actions together logically.

Source: callcenter.cpp

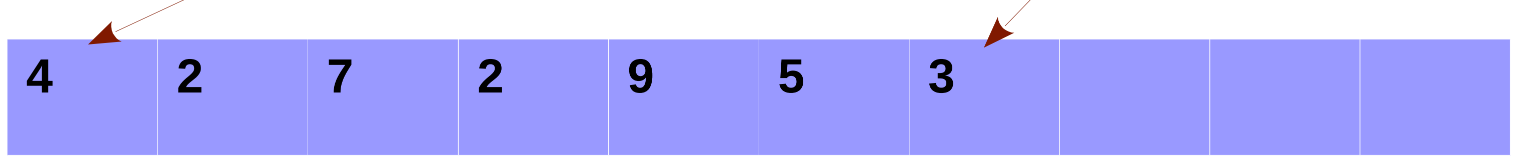
Queue Implementation

Recall circular list

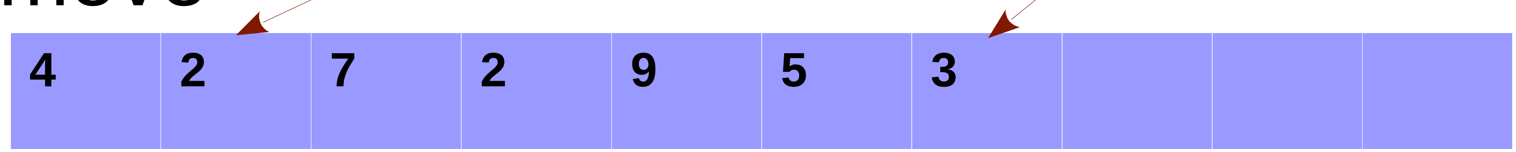
- This time, we will use arrays.



- Insert 3

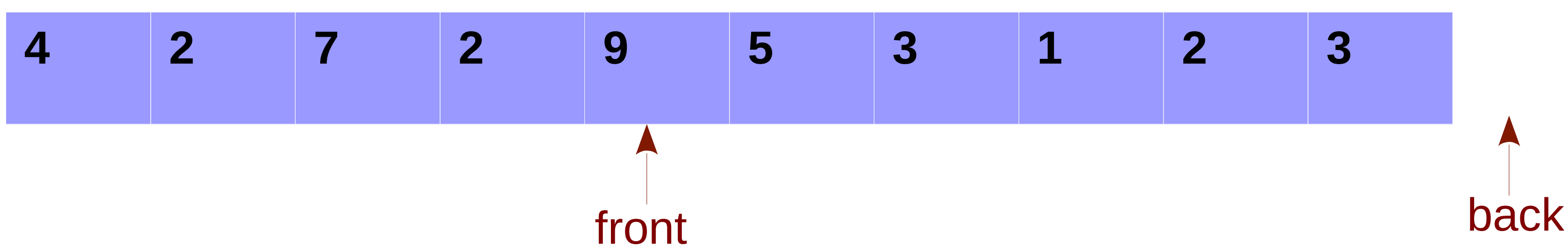


- Remove



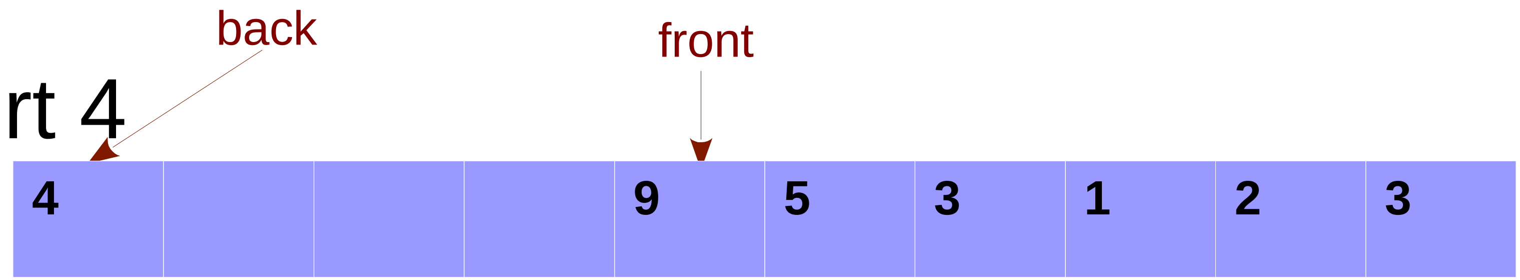
qimpl.c

- Remove, Remove, Remove, Insert 1, 2, 3, 4

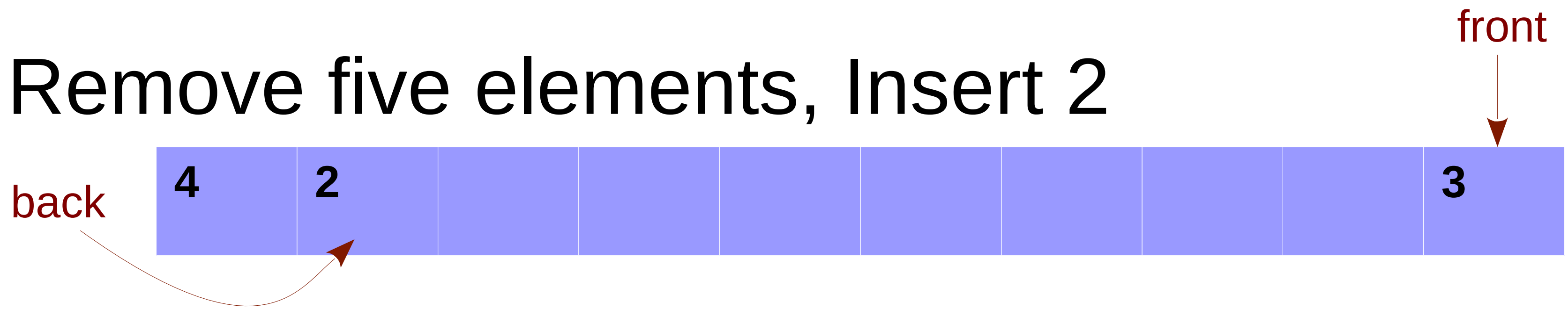


Wrap-around

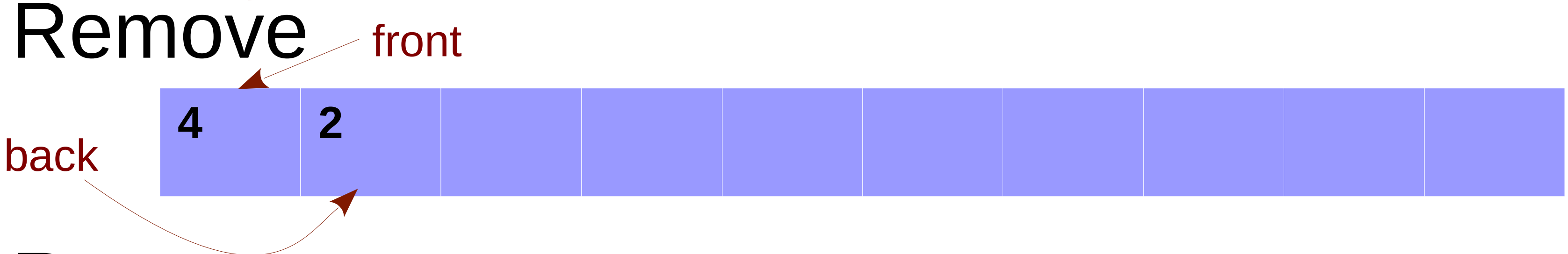
- Insert 4



- Remove five elements, Insert 2

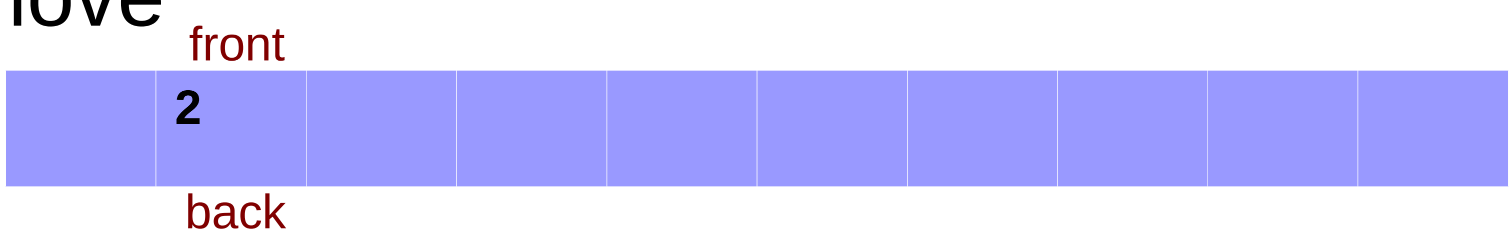


- Remove

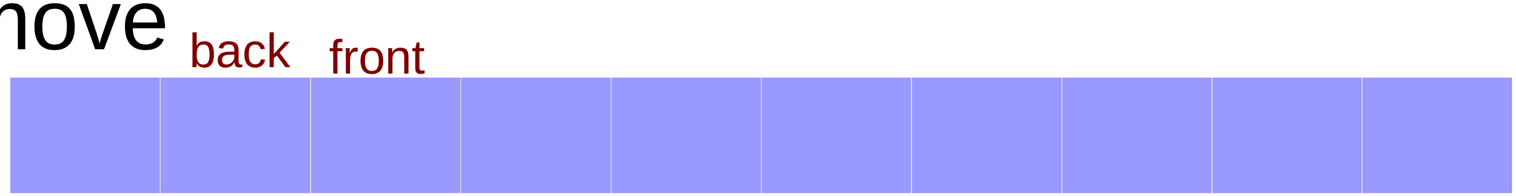


qimpl2.c

- Remove



- Remove



Queue Conditions

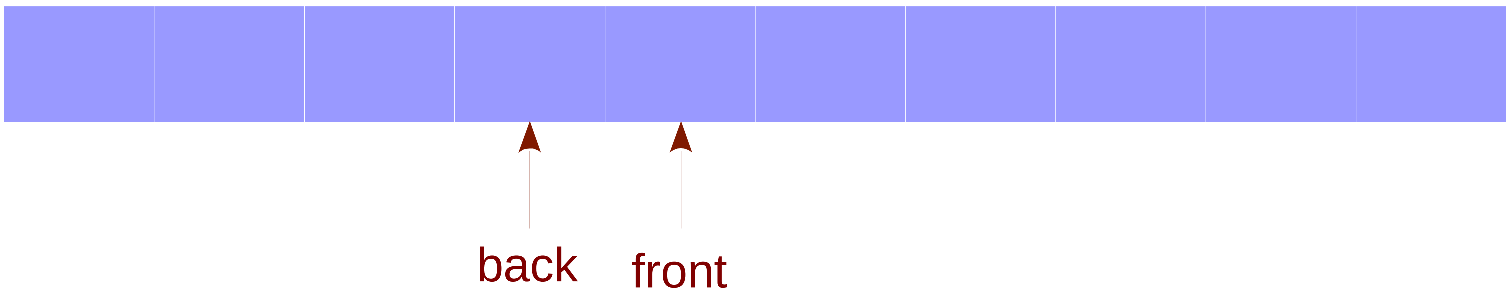
- Queue is empty:
  - when $\mathrm{front} > \mathrm{back}$ (in previous slide)
  - That is also initialization: $\mathrm{front} = 0$, $\mathrm{back} = -1$
  - Our implementation qimpl.c uses $\mathrm{front} = 0$, $\mathrm{back} = 0$
- Whichever you use, follow invariants:
  - qimpl.c: front points to the first element in the queue.

    back points to the place where next element should be inserted.
  - Previous slide: front points to the first element in the queue.

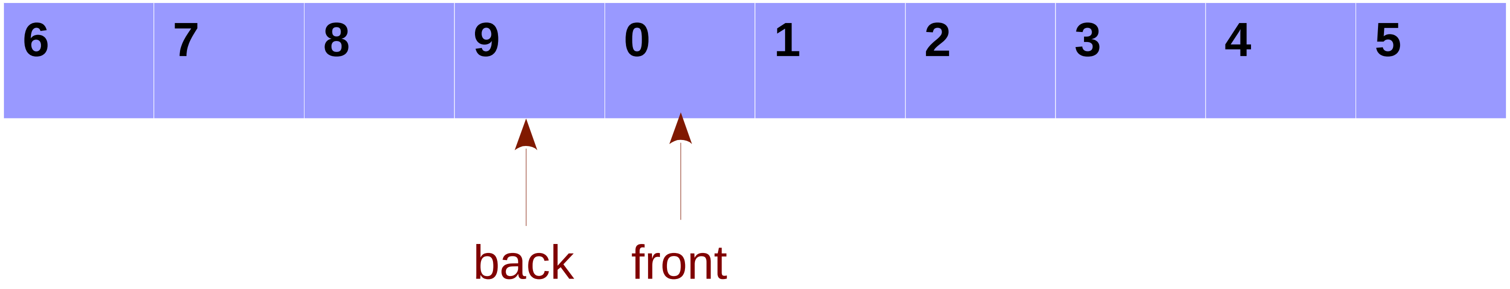
    back points to the last element in the queue.
- **Classwork**: Write conditions for when queue is full.

Empty versus Full

- Empty queue



- Full queue



- Possible solutions
  - Leave one space unused (N-1 elements).
  - Track size separately (used in qimpl.c).

Practice problems

- Implement a stack using two queues.
  - push/pop should be implemented using enqueue / dequeue.
- Implement a queue using two stacks.
- Implement three stacks using an array (without space wastage).
- Solve problems at the end of Chapter 3.

Learning Outcomes

- Use List, Stack, Queue ADTs in applications.
- Implement these ADTs using C/C++ with pointers or arrays.
- Study various applications using these data structures.