Trees

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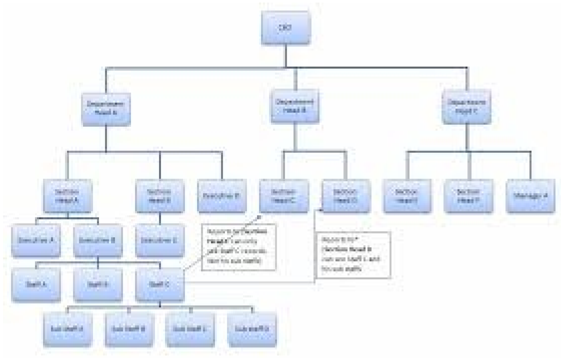
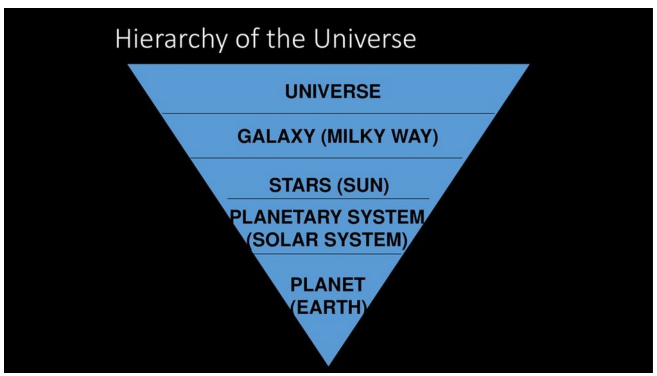
Manager-Employee Relation

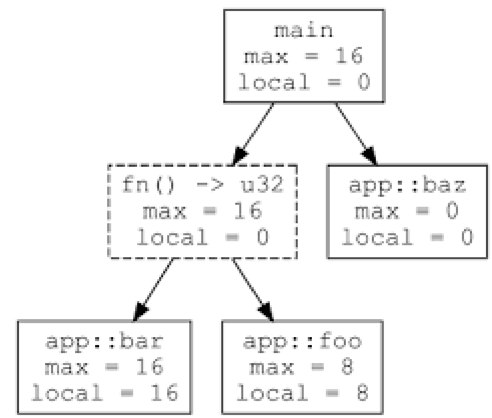


Google Maps

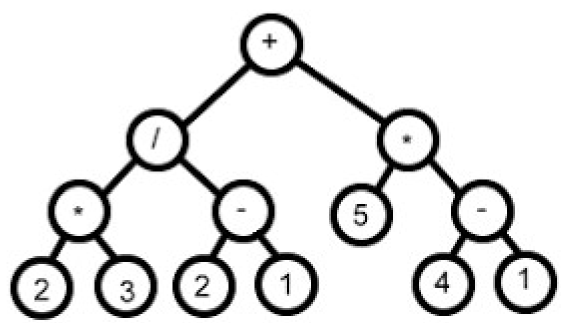


Planetary Hierarchy

Trees



Modeling Computation



Expression tree for 2*3/(2-1)+5*(4-1)

Expression Evaluation

Nomenclature

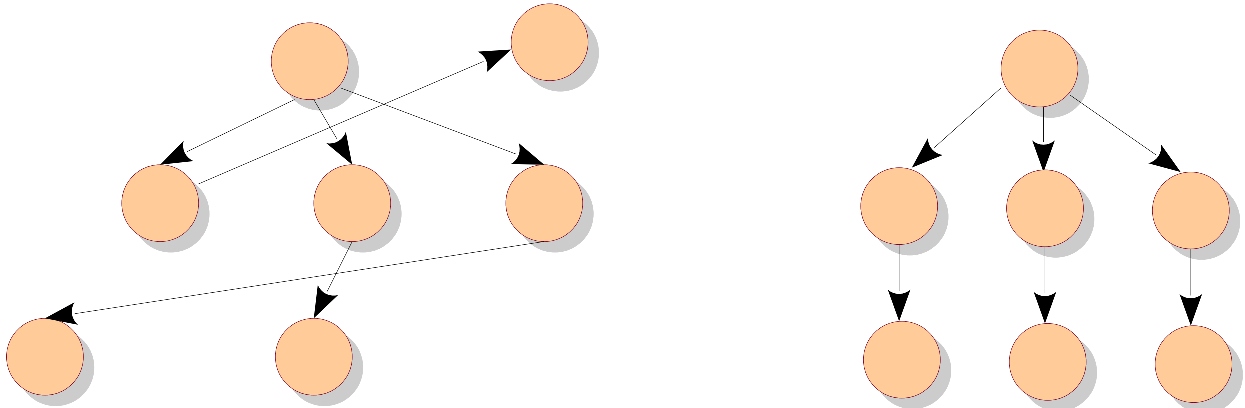
- Root
- Stem
- Branches
- Leaves
- ~~Fruits~~
- ~~Flowers~~

Edges (Stem, Branches)

Definition

A tree is a collection of nodes.
It could be empty. *// base case*

Otherwise, it contains a root node,
connected to zero or more (child) nodes,
each of which is a tree in itself! *// recursive*

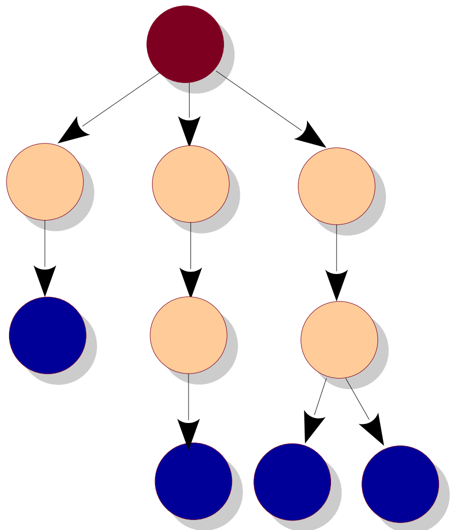


Alternatively, a tree is a collection of nodes and directed edges, such that each node except one has a single parent. The node without a parent node is the root.

Nomenclature

Root has no parent.
Leaves have no children.
Non-leaves are **internal nodes**.

Each node is *reachable* from the root.

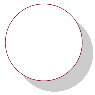


The whole tree can be accessed via root.

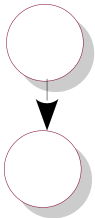
Each node can be viewed as the root of its unique *subtree*.

Empty Tree

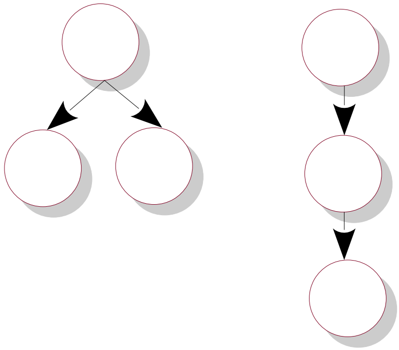
Tree with one node



Tree with two nodes

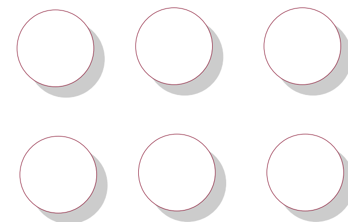


Trees with three nodes

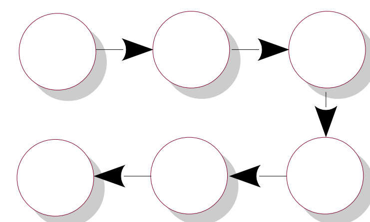


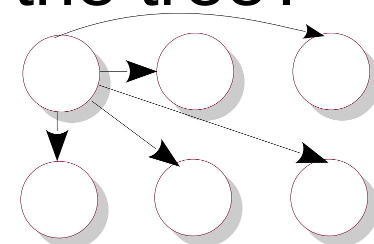
Properties

- A tree has six nodes.



  - What is the minimum number of edges in the tree?
  - What is the maximum?
  - Generalization for N nodes?

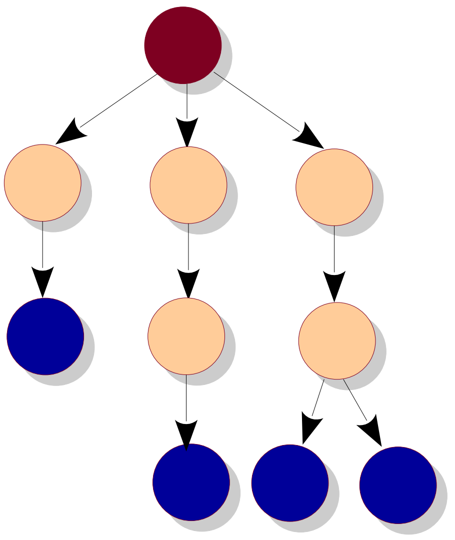




- How many (undirected) paths exist between two nodes?

More Nomenclature

- Sibling
  - What is the maximum number of siblings a node may have in an N node tree?
- Grandparent, grandchild
- Ancestor, descendant
- Path, length
- Height, depth



Exercises

- Given (a pointer to) a node in an employee tree, list all its direct and indirect subordinates.
- Same as above with the name of the employee given.
- Find distance between two nodes.
- Find tree diameter (max. distance).
- Convert infix to postfix (using a tree).
- Mirror a tree vertically.
- Find if there is a directed path from p to q.

Learning Outcomes

- Apply tree data structure in relevant applications.
- Construct trees in C++ and perform operations such as insert.
- Perform traversals on trees.
- Analyze complexity of various operations.

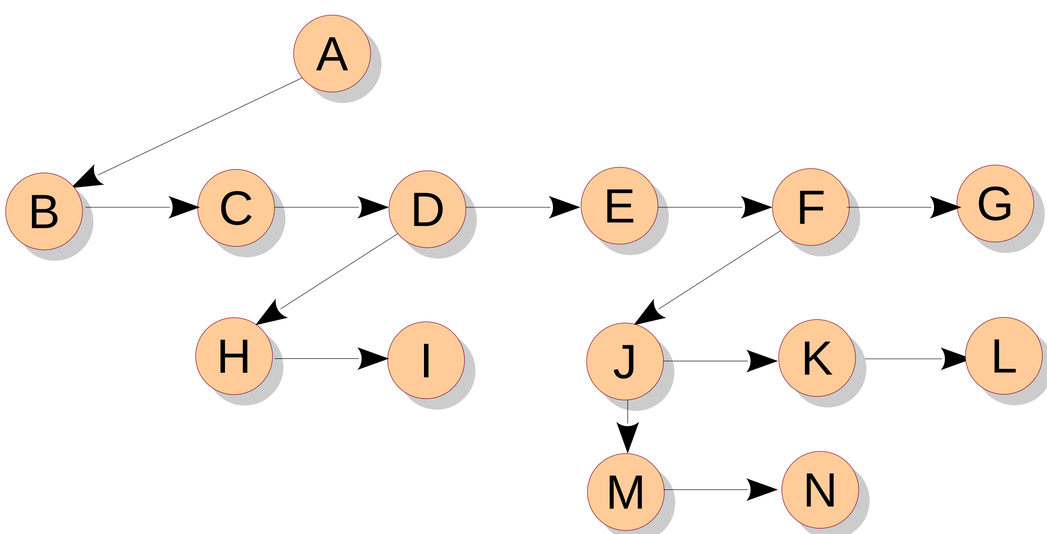
Implementation

- A challenge is that the maximum number of children is unknown, and may vary dynamically.

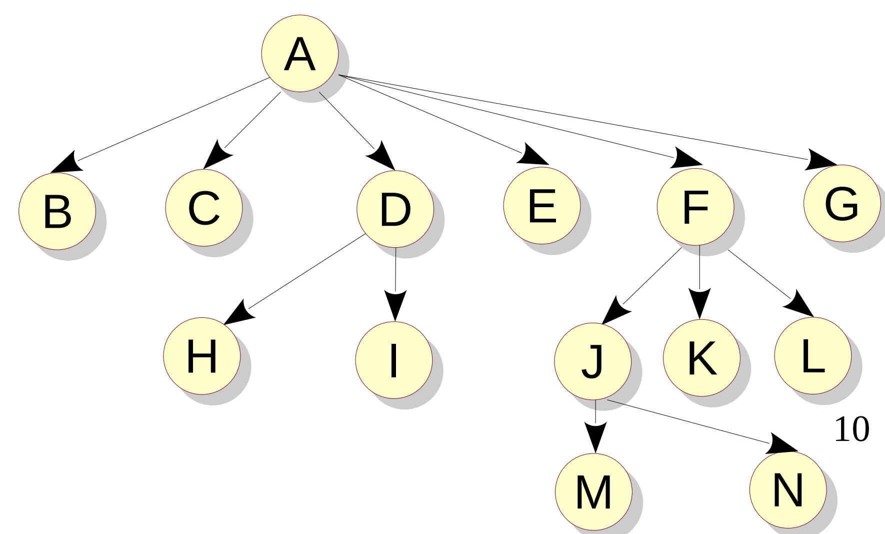
```c
typedef struct TreeNode *PtrToNode;

struct TreeNode {
        char data;
        PtrToNode firstChild;
        PtrToNode nextSibling;
};
```

```cpp
#include <vector>
typedef struct TreeNode *PtrToNode;

struct TreeNode {
        char data;
        std::vector<PtrToNode> children;
};
```

C

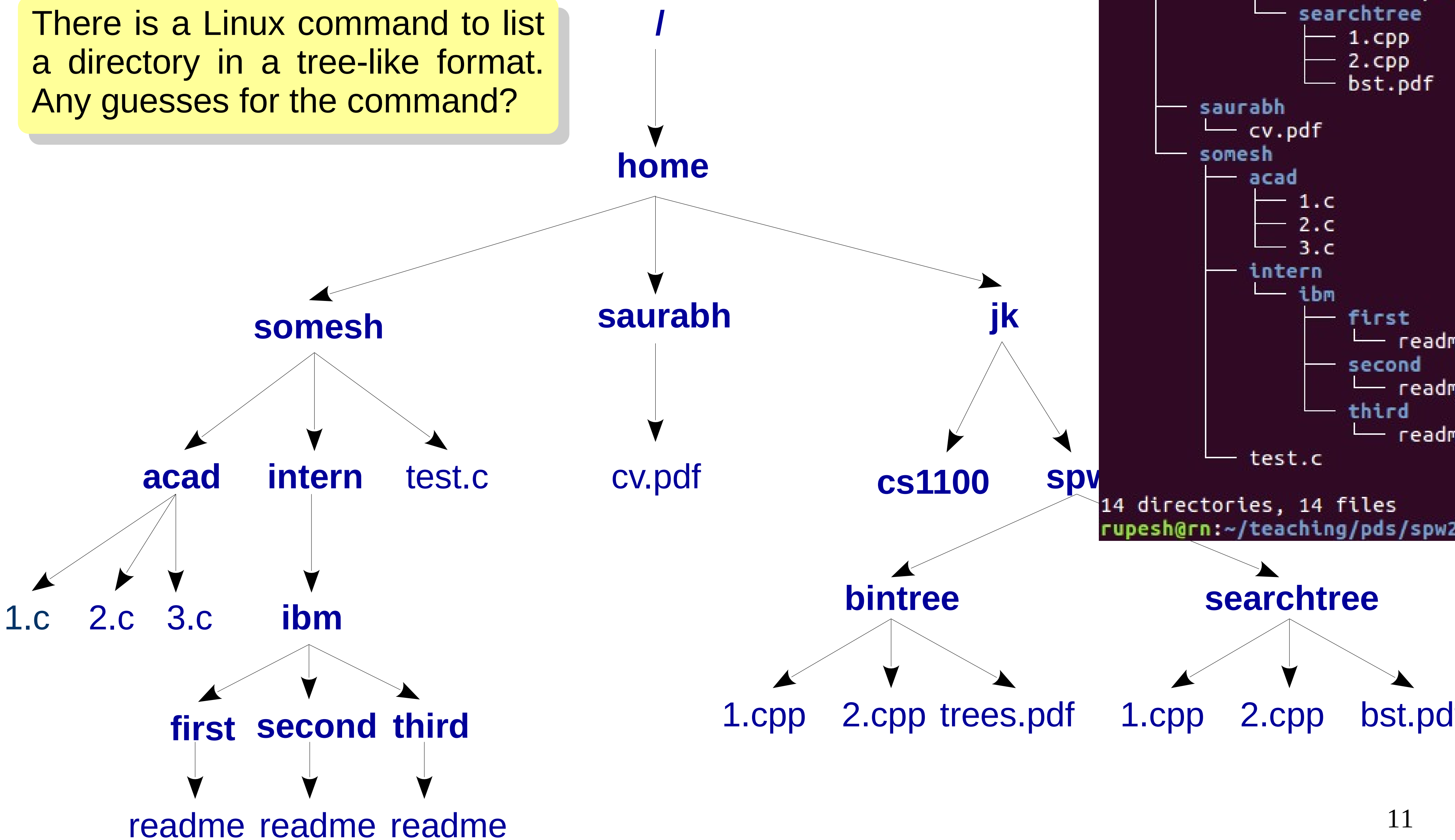


C++



Directory Listing

There is a Linux command to list a directory in a tree-like format. Any guesses for the command?




```
rupesh@rn:~/teaching/pds/spw2019
.
└── home
    ├── jk
    │   ├── cs1100
    │   └── spw
    │       ├── bintree
    │       │   ├── 1.cpp
    │       │   ├── 2.cpp
    │       │   └── trees.pdf
    │       └── searchtree
    │           ├── 1.cpp
    │           ├── 2.cpp
    │           └── bst.pdf
    ├── saurabh
    │   └── cv.pdf
    └── somesh
        ├── acad
        │   ├── 1.c
        │   ├── 2.c
        │   └── 3.c
        ├── intern
        │   └── ibm
        │       ├── first
        │       │   └── readme
        │       ├── second
        │       │   └── readme
        │       └── third
        │           └── readme
        └── test.c

14 directories, 14 files
rupesh@rn:~/teaching/pds/spw2019
```

Switch to code.

2.cpp and 3.cpp

Traversals

- Preorder
  - Process each node <u>before</u> processing its children.
  - Children can be processed in any order.
- Postorder
  - Process each node <u>after</u> processing its children.
  - Children can be processed in any order.
- Preorder and postorder are examples of Depth-First Traversal.
  - Children of a node are processed <u>before</u> processing its <u>siblings</u>.
  - The other way is called Breadth-First or Level-Order Traversal.

Preorder

Iterative

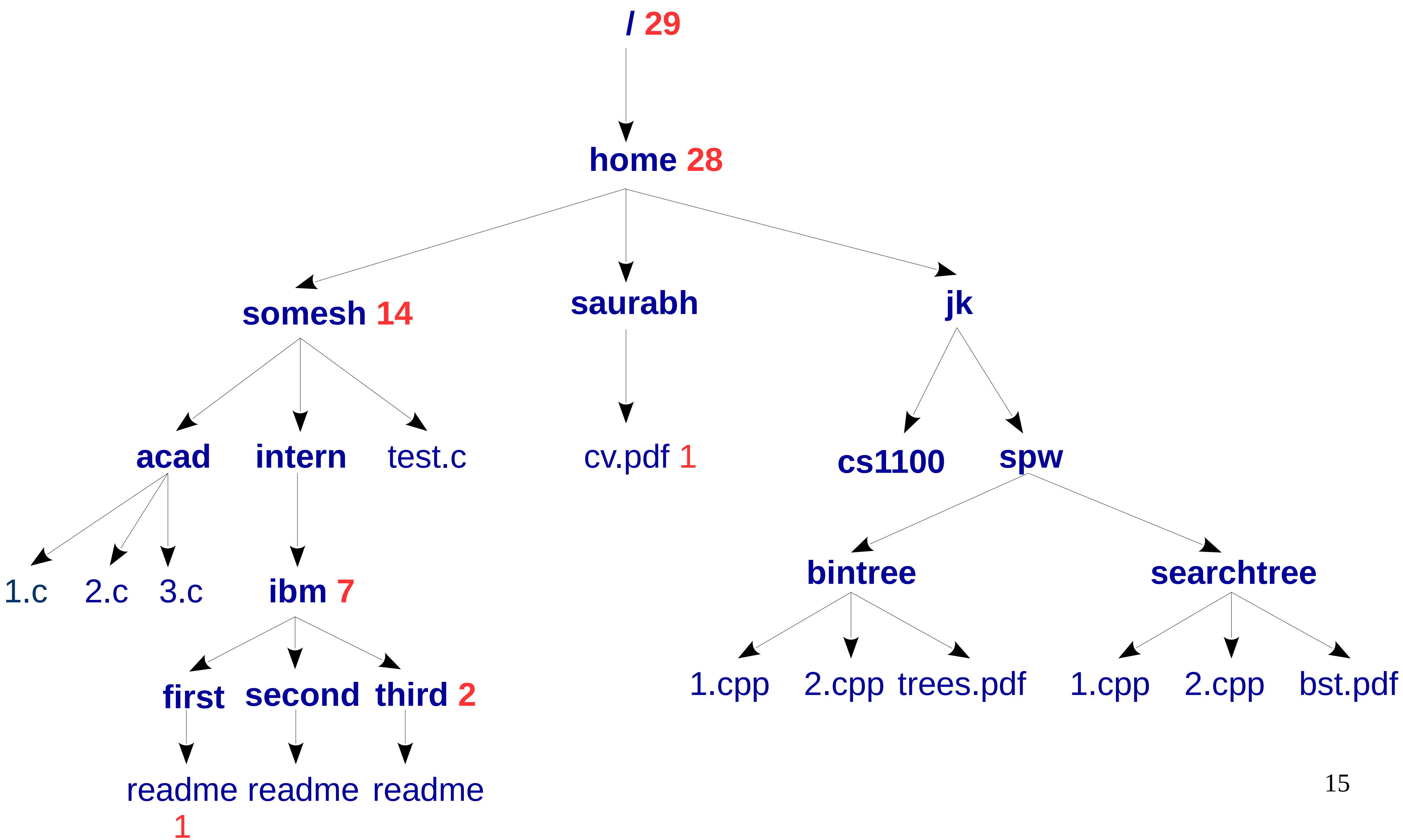
```
void Tree::preorder() {
        std::stack<PtrToNode> stack;
        stack.push(root);

        while (!stack.empty()) {
                PtrToNode rr = stack.top();
                stack.pop();
                if (rr) {
                        rr->print();
                        for (auto child: rr->children)
                            stack.push(child);
                }
        }
}
```

Recursive

```
void Tree::preorder(PtrToNode rr) {
        if (rr) {
                rr->print();
                for (auto child:rr->children)
                    preorder(child);
        }
}
void Tree::preorder() {
        preorder(root);
}
```

Switch to code: 4.cpp, 6.cpp
Classwork: Indent files as per their depth. What is the code complexity? Note that indentation time also needs to be considered.

Find full size of each directory



Postorder

Iterative

Try it out offline.

Recursive

```
void Tree::postorder(PtrToNode rr) {
	if (rr) {
		for (auto child:rr->children)
			postorder(child);
		rr->print();
	}
}
void Tree::postorder() {
	postorder(root);
}
```

Switch to code: 5.cpp

Story so far...

- General trees
  - arbitrary number of children
  - Resembles several situations such as employees, files, ...
- Special trees
  - Fixed / bounded number of children
  - Resembles situations such as expressions, boolean flows, …
  - All the children may not be present.

K-ary Trees

```
typedef struct TreeNode *PtrToNode;

struct TreeNode {
	int data;
	PtrToNode firstChild;
	PtrToNode nextSibling;
};
```

```
#include <vector>
typedef struct TreeNode *PtrToNode;

struct TreeNode {
	int data;
	std::vector<PtrToNode> children;
};
```

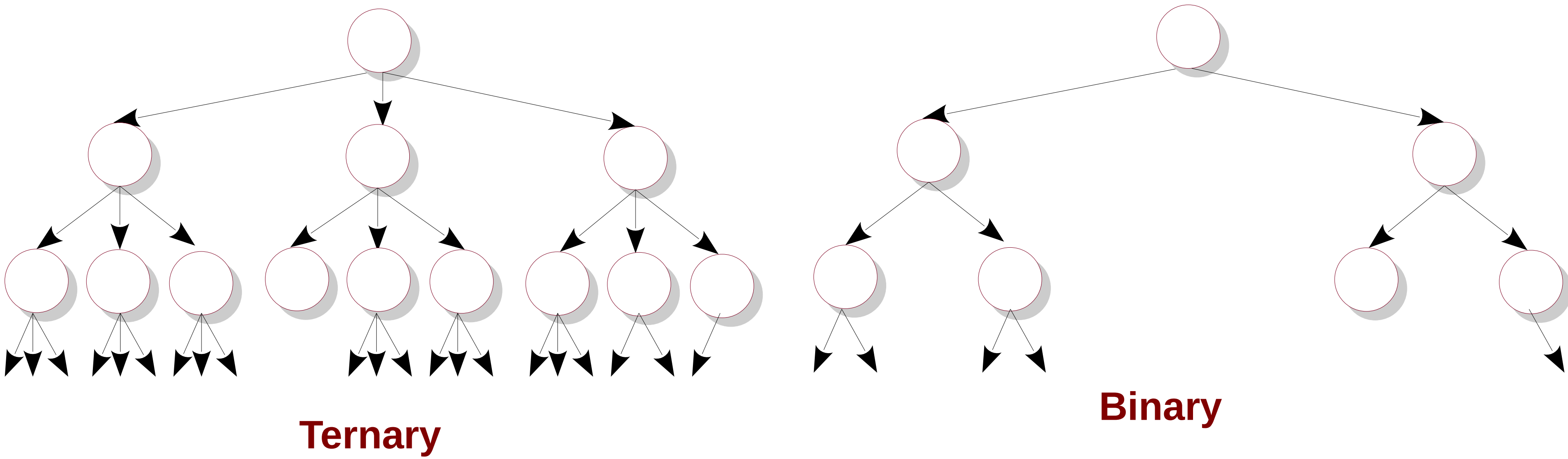
For a fixed K

```
typedef struct TreeNode *PtrToNode;

struct TreeNode {
	int data;
	PtrToNode children[K];
};
```

When K == 2

```
typedef struct TreeNode *PtrToNode;

struct TreeNode {
	int data;
	PtrToNode left;
	PtrToNode right;
};
```

K-ary Trees



For a fixed K

```
typedef struct TreeNode *PtrToNode;

struct TreeNode {
        int data;
        PtrToNode children[K];
};
```

When K == 2

```
typedef struct TreeNode *PtrToNode;

struct TreeNode {
        int data;
        PtrToNode left;
        PtrToNode right;
};
```

Properties of Binary Trees

- For an N node binary tree (N > 0):
  - What is the maximum height? **N**
  - What is the minimum height? **$\log_2(N)$**
  - How many NULL pointers? **N + 1**
  - How many min/max leaves? **0/1, N / 2**
- What is the maximum number of nodes a binary tree of height H may have? **$2^H - 1$**
- Full nodes (nodes with two children):
  - how many minimum, maximum? **0, N / 2 - 1**
- Show that #full nodes + 1 == #leaves in a non-empty binary tree.

Operations on Trees

- **Insert**: our addChild would take care of this.
  - Given pointers, this is constant time operation.
- **Remove**: Update parent's pointer to NULL (and free memory).
  - What if the node getting removed has children?
  - Based on the above answer, the complexity could be O(1) or O(N)
- **Search**: Our tree traversals can help.
  - Can a tree contain duplicate values?
  - This is O(N), since the whole tree needs to be searched in the worst case.

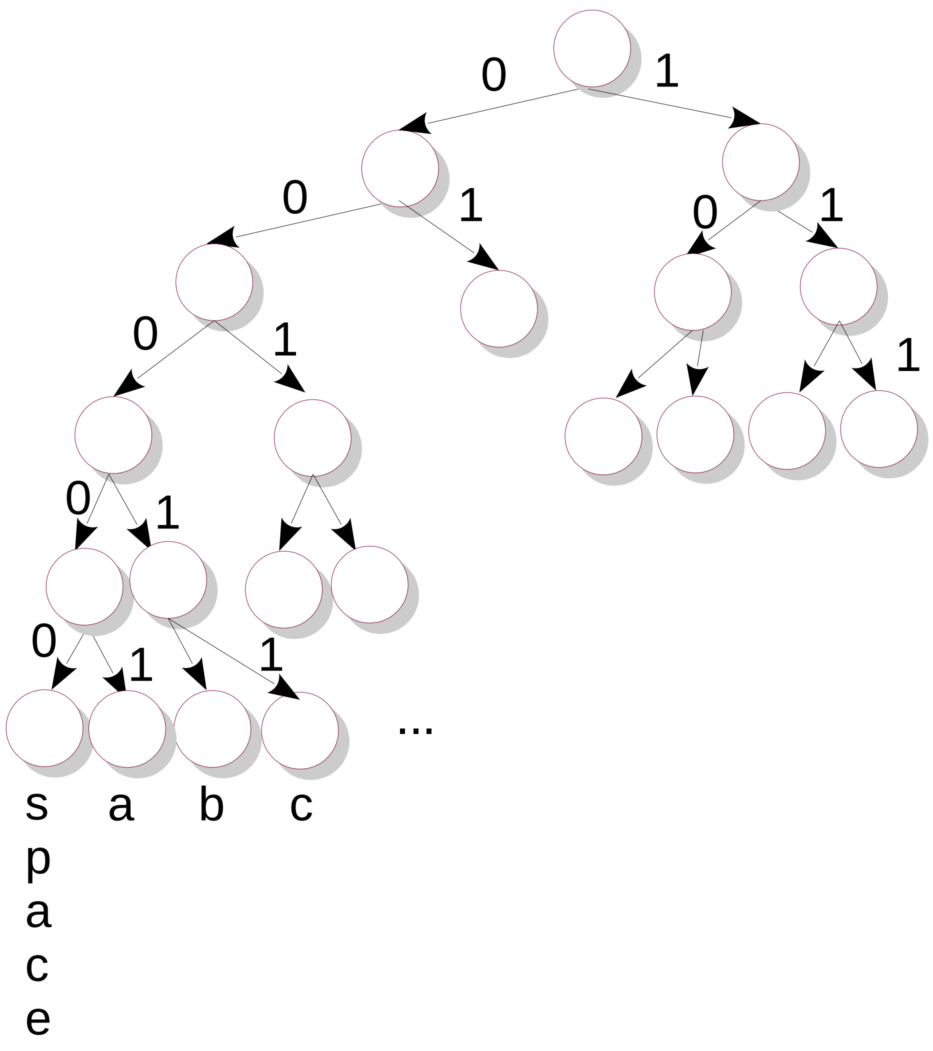
Some Questions?

- What if a child node is common to two parents?
  - Ancestry
- Can the edge be undirected?
- Can the edges have weights?
- Can there be multiple roots?
- Can there be multiple edges between two nodes?
- Is it okay to draw a tree with root at the bottom?

Coding (a little different)

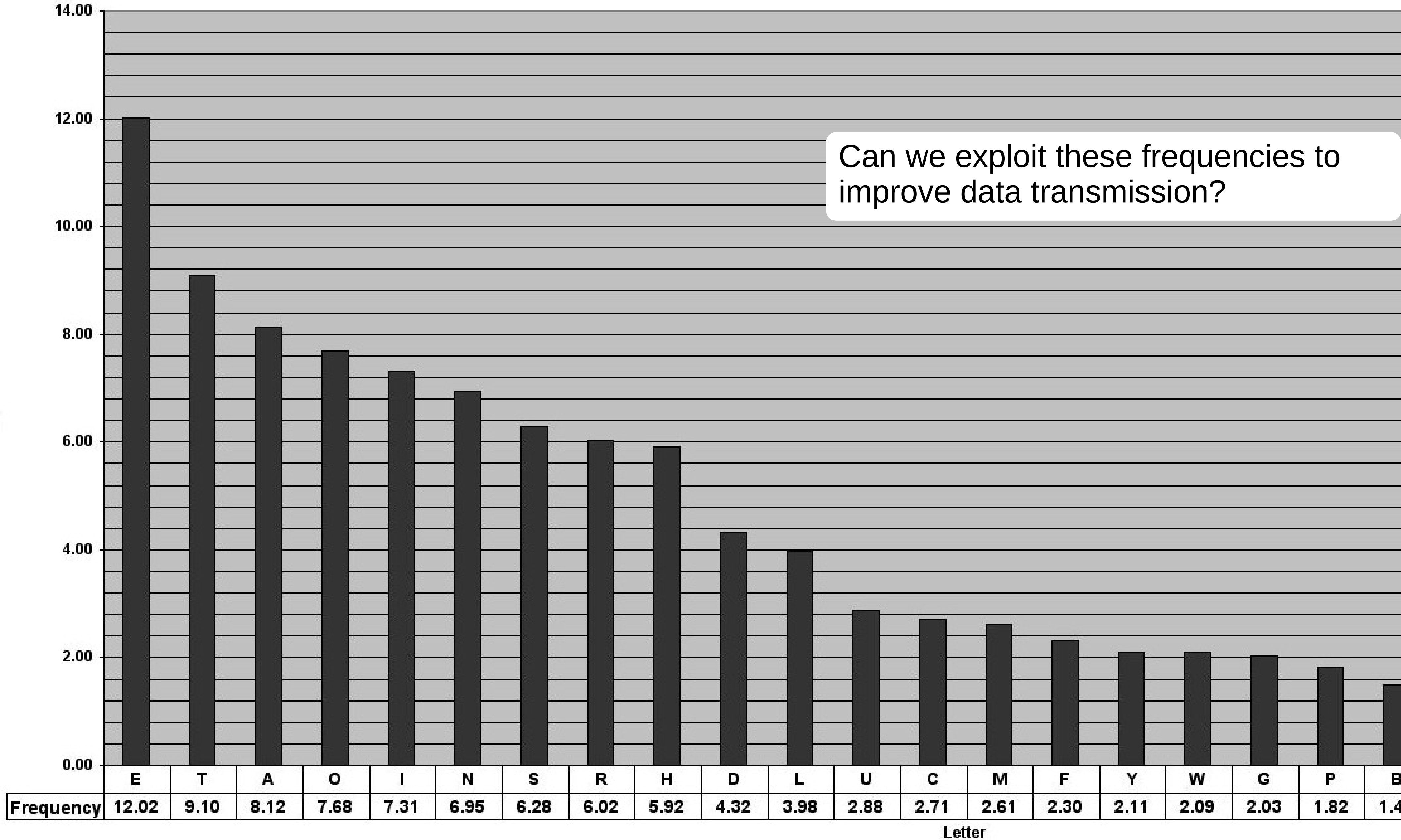
- I want to transmit some data.
- Data contains a-z and space.
- For these 27 characters, I need 5 bits.
  - For N characters, I need $\log_2(N)$ bits.
- Encoding pattern:
  - space = 00000, a = 00001, b = 00010, …, z = 11010
- Decoding:
  - Each 5-bit string represents a unique character (except the last five strings: 11011 to 11111).
  - What is 00100110010111000001011010111 1?

How is a code related to a tree?



- It is a binary tree.
- Tree is (almost) complete.
- Has height of 5, equal to the code length.
- Each character has a **unique** code, because each tree node has a unique path from the root.
- **Encoding**: Given a character, traverse back from its node towards the root, and we get the reverse of its code.
- **Decoding**: Given a code, traverse the tree from the root, and the node we reach is the corresponding character.
- None of the interior nodes represents a character.

What is this plot?



Can we exploit these frequencies to improve data transmission?

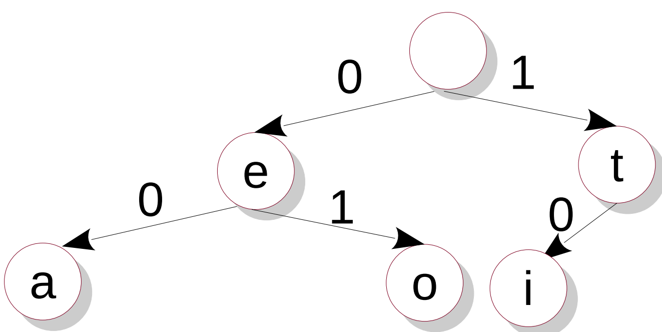
Source: http://pi.math.cornell.edu/~mec/2003-2004/cryptography/subs/frequencies.html

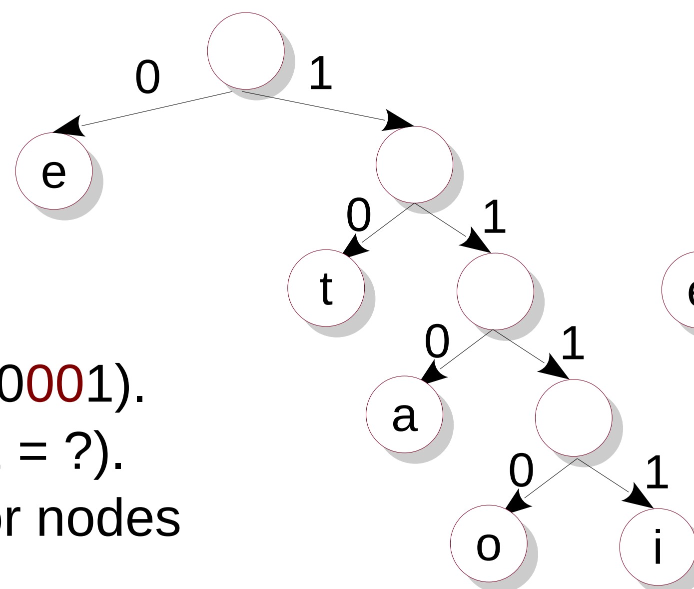
Make the common case faster!

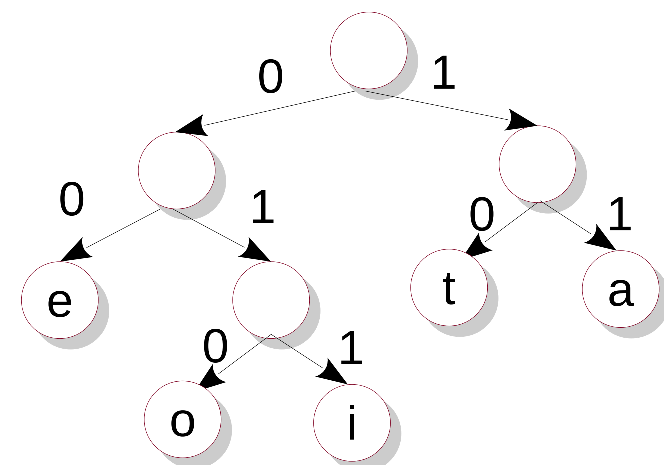
- If most people order vanilla ice-cream, keep it in front.
- If only a few students buy fish, keep it at a separate counter.
- If most people coming to the department use bicycle, bicycle parking should be prioritized.
- If most of the humans in the classroom stay in hostels, the classes should be held in hostels!
- If ‘e’ gets used more often, can we transmit it faster?

Shorter Codes

Character	Frequency	Code	Code 2	Code 3
e	12.02	0	0	00
t	9.10	1	10	10
a	8.12	00	110	11
o	7.68	01	1110	010
i	7.31	10	1111	011







Encoding is easy (e.g., eat = 0001).
Decoding is tough (e.g., 0001 = ?).
This happens because interior nodes also represent data.
We need data only at the leaves.

Called prefix codes.
We can 001110 easily now.

Prefix Codes

- Such codes were invented by Huffman.
  - as a term paper at MIT during his PhD.
  - had the habit of keeping poisonous snakes as pets.
- Prefix codes are easy to decode.
  - No ambiguous decoding possible.
- Faster transmission of frequent data.
  - In practice, close to 40-50% improvement
- We will study Huffman’s algorithm during Heaps.

Learning Outcomes

- Apply tree data structure in relevant applications.
- Construct trees in C++ and perform operations such as insert.
- Perform traversals on trees.
- Analyze complexity of various operations.