CS6015: Linear Algebra and Random Processes
Quiz - 2
Course Instructor : Prashanth L.A.
Date : Aug-28, 2017 **Duration** : 30 minutes

Name of the student :
Roll No :

INSTRUCTIONS: For true/false questions, you do not have to justify the answer. For the rest, provide proper justification for the answers. Please use rough sheets for any calculations *if necessary.* Please **DO NOT** submit the rough sheets. DO NOT use pencil for writing the answers.

1. True or False? Answer any five. $(2+2+2+2+2$ marks$)$

   Note: 2 marks for the correct answer and -1 *for the wrong answer.*

   (a) If the columns of a matrix A are linearly dependent, then $Ax = 0$ has a non-trivial solution.

   (b) Let S be a subspace of $\mathbb{R}^n$. The projection p of a $b \in \mathbb{R}^n$ is zero if and only if
   $$b^{\mathsf{T}} y = 0 \text{ for all } y \in S.$$

   (c) A matrix A can have a column space that contains $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ and a null space that contains $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

   (d) There exists a $m \times n$ matrix A with $m < n$ whose null space is $\{0\}$.

   (e) The dimensions of the row space and column space of a $m \times n$ matrix, with $m \neq n$, are the same.

   (f) If P is a projection matrix, then $(I - P)^2 = I - P$.

   (g) If A is a $m \times r$ matrix with r independent columns and B is a $r \times n$ matrix with r independent rows, then AB is invertible.

2. For each of the matrices below, solve $Ax = 0$ and characterize the null space by finding its basis.

   (a) $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ (2 marks)

   (b) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ (5 marks)

   (c) $A = \begin{bmatrix} 1 & 2 & \dots & n \\ 2 & 3 & \dots & n+1 \\ \vdots & \vdots & \ddots & \vdots \\ n & n+1 & \dots & 2n-1 \end{bmatrix}$ (3 marks)