CS5691: Pattern recognition and machine learning
Quiz - 1
Course Instructor : Prashanth L. A.
Date : Feb-1, 2019 **Duration** : 30 minutes

Name of the student :
Roll No :

INSTRUCTIONS: For MCQ questions, you do not have to justify the answer. For the rest, provide proper justification for the answers. Please use rough sheets for any calculations *if necessary.* Please **DO NOT** submit the rough sheets. DO NOT use pencil for writing the answers.

I. Multiple Choice Questions

Note: 1 mark for the correct answer. Only one answer is correct. Please write the choice code a, b, c or d in the answer box provided.

(1) Suppose X is uniformly distributed over $[0,5]$ and Y is uniformly distributed over $[0,4]$. If X and Y are independent, then $\mathbb{P}(\max(X,Y) > 3)$ is

(a) $\frac{9}{20}$
(b) $\frac{1}{20}$
(c) $\frac{11}{20}$
(d) 1

Answer: ☐

(2) Let X_i, $i = 1, \ldots, 4$ be independent Bernoulli r.v.s each with mean $p = 0.1$ and let $S = \sum_{i=1}^{4} X_i$. Then,

(a) $\mathbb{E}(X_1 \mid S = 2) = 0.1$.
(b) $\mathbb{E}(X_1 \mid S = 2) = 0.5$.
(c) $\mathbb{E}(X_1 \mid S = 2) = 0.25$.
(d) $\mathbb{E}(X_1 \mid S = 2) = 0.75$.

Answer: ☐

(3) Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, and $v_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Let $C(A)$ and $N(A)$ denote the column and null space, respectively of any matrix A. Then, which of the following statements is **false**?

(a) $v_1, v_2 \in C(A)$, and $v_3 \in N(A)$ for some matrix A.
(b) $v_1, v_2 \in C(A)$, and $v_3, v_4 \in N(A)$ for some matrix A.
(c) $v_1 \in C(A)$, and $v_3, v_4 \in N(A)$ for some matrix A.
(d) $v_1 \in C(A)$, and $v_3 \in N(A)$ for some matrix A.

Answer: ☐

(4) Let $Z = (X, Y)$ be a bivariate normal random variable. Then, which of the following statements is **false**?

(a) X and Y are independent if and only if they are uncorrelated.

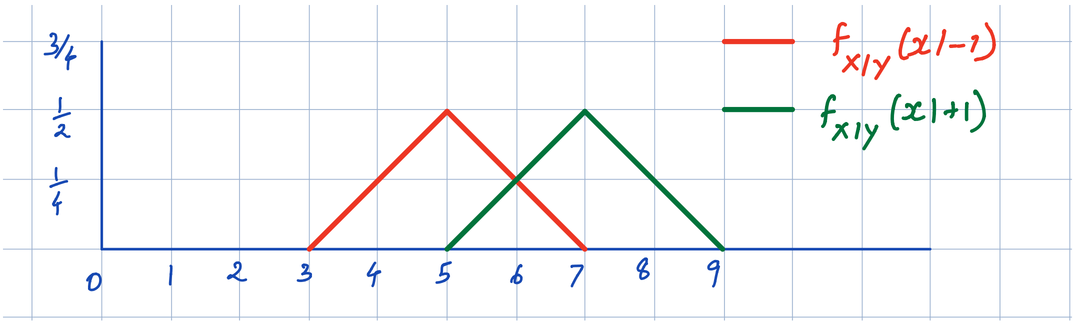
(b) $X + Y$ is univariate normal.

(c) $Y \mid X = x$ is distributed as a normal random variable.

(d) $X + Y$ and $X - Y$ are independent.

Answer: ☐

(5) Let $P(Y = -1) = \frac{1}{3}$, and $P(Y = +1) = \frac{2}{3}$. The class-conditionals $P(X|Y)$ are given by the graph below. The Bayes classifier is then given by which option below? (Triangle on left is the class conditional for $Y = -1$).



(a) $h^*(x) = \begin{cases} -1 & \text{if } x \leq 6 \\ +1 & \text{if } x > 6 \end{cases}$

(b) $h^*(x) = \begin{cases} -1 & \text{if } x \leq \frac{17}{3} \\ +1 & \text{if } x > \frac{17}{3} \end{cases}$

(c) $h^*(x) = \begin{cases} -1 & \text{if } x \leq \frac{19}{3} \\ +1 & \text{if } x > \frac{19}{3} \end{cases}$

(d) $h^*(x) = \begin{cases} -1 & \text{if } x > 6 \\ +1 & \text{if } x \leq 6 \end{cases}$

Answer: ☐

II. A problem that requires a detailed solution

(1) Let X and Y be r.v.s with the joint density given by

$$f(x, y) = \frac{1}{8\sqrt{3}\pi} \exp\left(-\frac{x^2}{6} - \frac{y^2}{24} + \frac{xy}{12} + \frac{x}{12} + \frac{y}{6} - \frac{7}{24}\right).$$

Answer the following: (3+2 marks)

(a) Find the means and variances of X and Y. Also, find the covariance between X and Y.

(b) Find the conditional density of Y given $X = x$. Also, calculate $\mathbb{E}[Y \mid x]$.