CS6130 : Advanced Graph Algorithms

Extensions of Bipartite Matchings and Max flows.

Homework Problem

Input : $(S \cup \mathcal{C}, E)$ → $(s, c) \in E \Rightarrow$ s is interested in c and c can take s.

S → students, $\mathcal{C}$ → courses

Quota : for every $c \in \mathcal{C}$ $q(c) =$ max # of students c can accomodate

Additional constraints : for every $c \in \mathcal{C}$

$N(c) = P_1 \,\dot\cup\, P_2 \,\dot\cup\, P_3 \dots \dot\cup\, P_k$

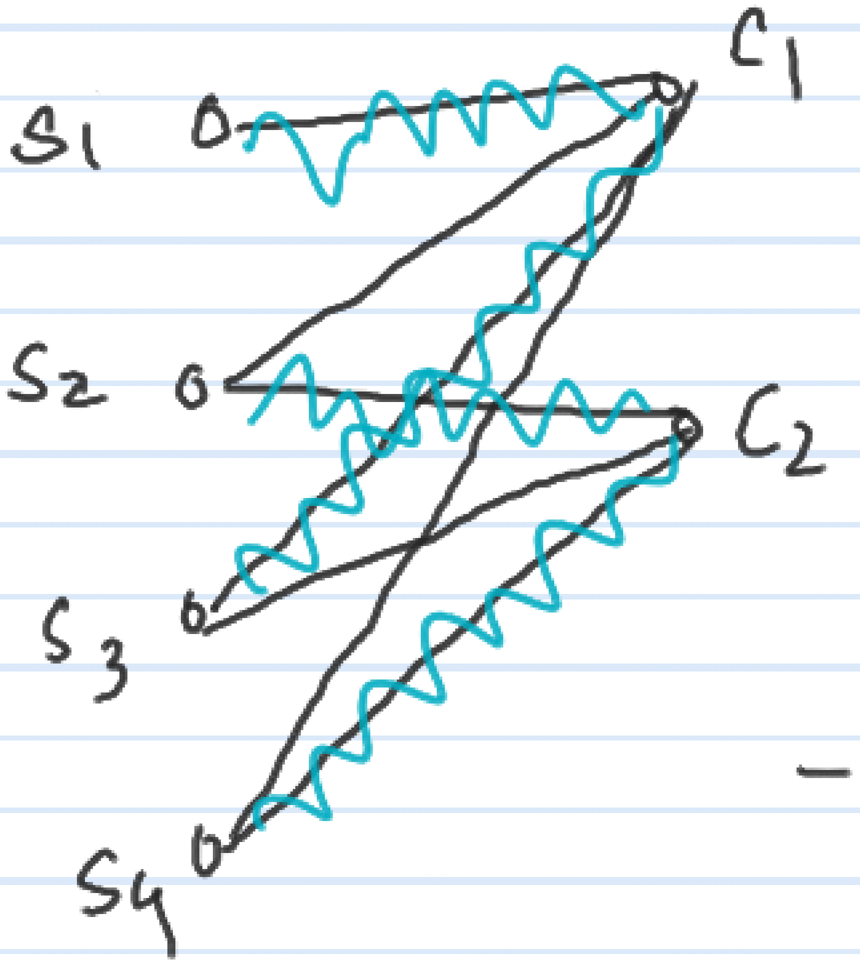
adjacent students are partitioned and each partition has its individual quota

$q(c, P_i)$: max # of students c can acco. from P_i

Goal: Compute a max matching respecting constraints

An Example:

$q(c_1) = 2$, $q(c_2) = 2$



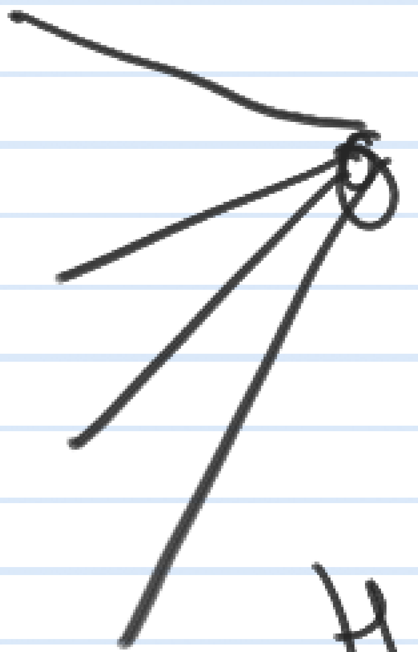
$P_1 = \{s_1, s_2\}$ $q=1$ | $P_1' = \{s_1, s_2, s_3\}$ $q=1$

$P_2 = \{s_3\ s_4\}$ $q=1$ | $P_2' = \{s_4\}$ $q=1$

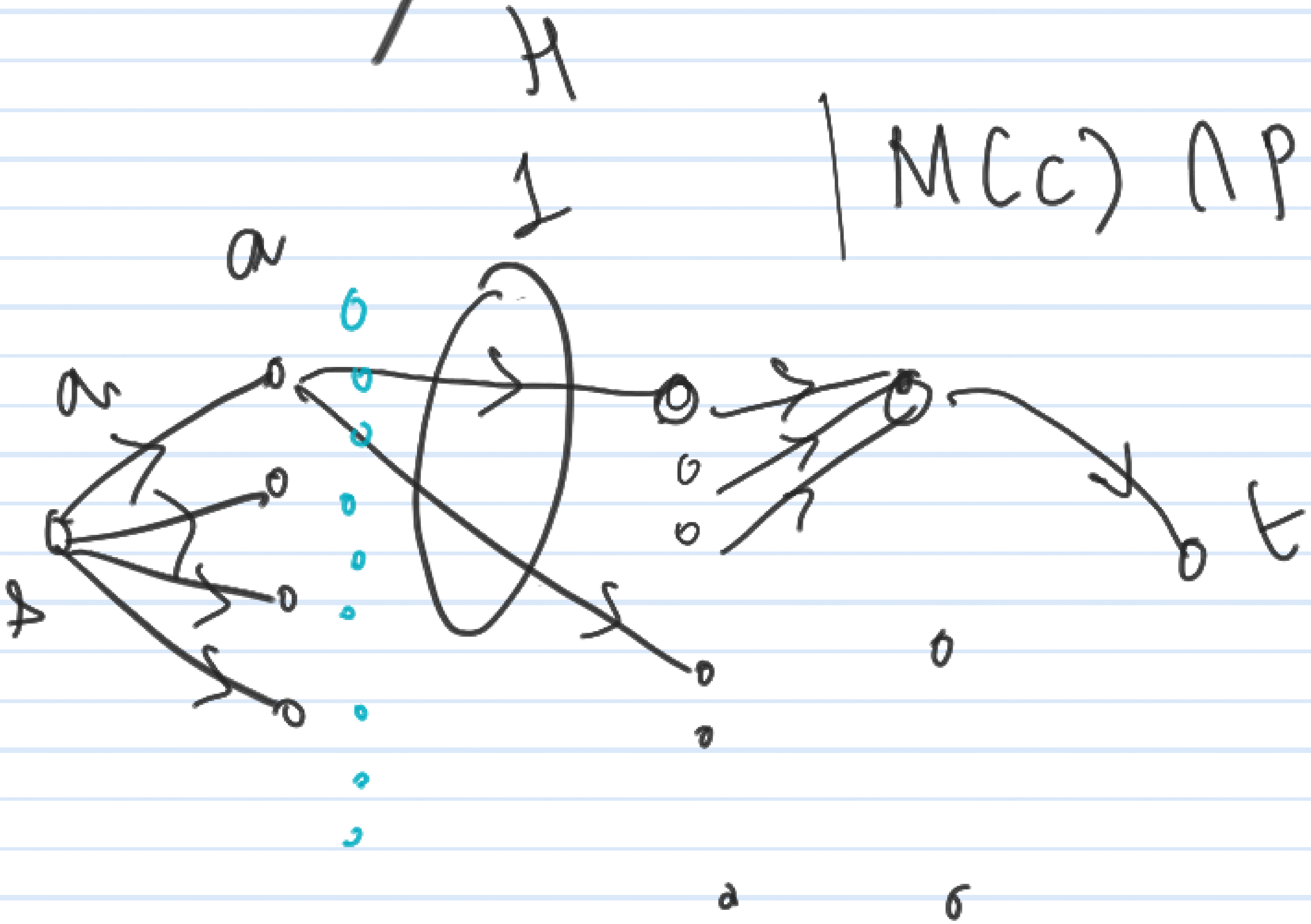
- in this example we cannot assign s_1 and s_2 to c_1 because of P_1 and its quota 1
- similarly, we cannot assign s_1 and s_3 to c_2 because of P_1' and its quota 1
- A possible assignment is $M = \{(s_1, c_1)(s_2\ c_2)\ (s_4, c_2)\}$ → is this valid / maximum?

$$N(C) = P_1 \,\dot{\cup}\, P_2 \,\dot{\cup}\, P_3 \ldots \dot{\cup}\, P_k$$

$$P_1 \downarrow a(P_1^c) \qquad P_2 \to a(P_2^c)$$



$$|M(C) \cap P_1| \le a(P_1^c)$$



Max flow ~~in~~ H corresp to a max mat

constraints on courses no longer form a partition and can be overlapping

$$|M(c) \cap P_i^c| \le q(P_i^c) \quad P_1^c, P_2^c \ldots P_k^c$$

$$P_i \cap P_j = \phi$$

Bipartite matching with classification constraints $\rightarrow$ Flows.

Demands and Circulation

Input : Directed n/w with $c(e) > 0$

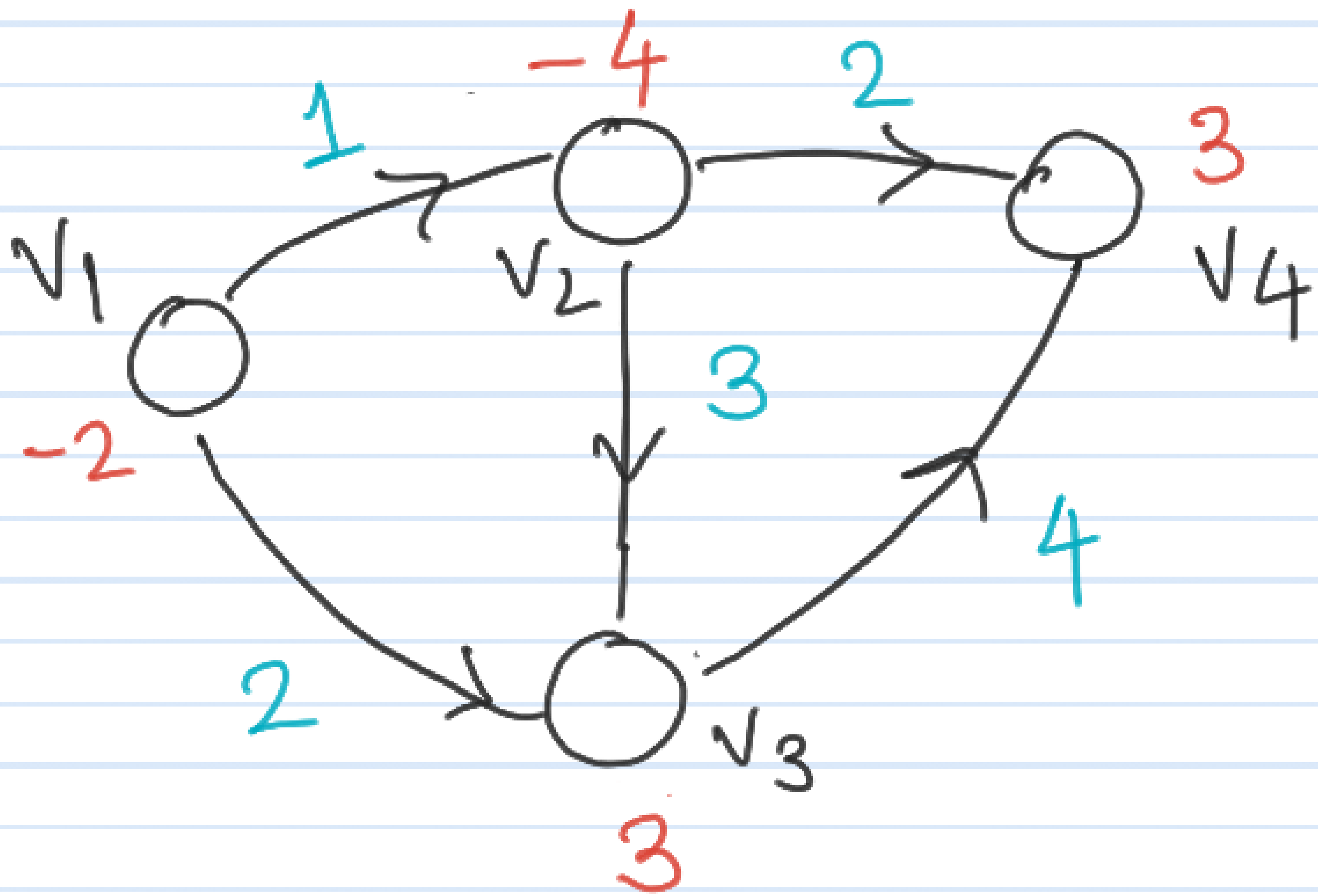
for every v $d(v)$ → can be $>$ $=$ $<$ 0

no designated source and target vertex

Goal : Compute A flow function $f(e)$

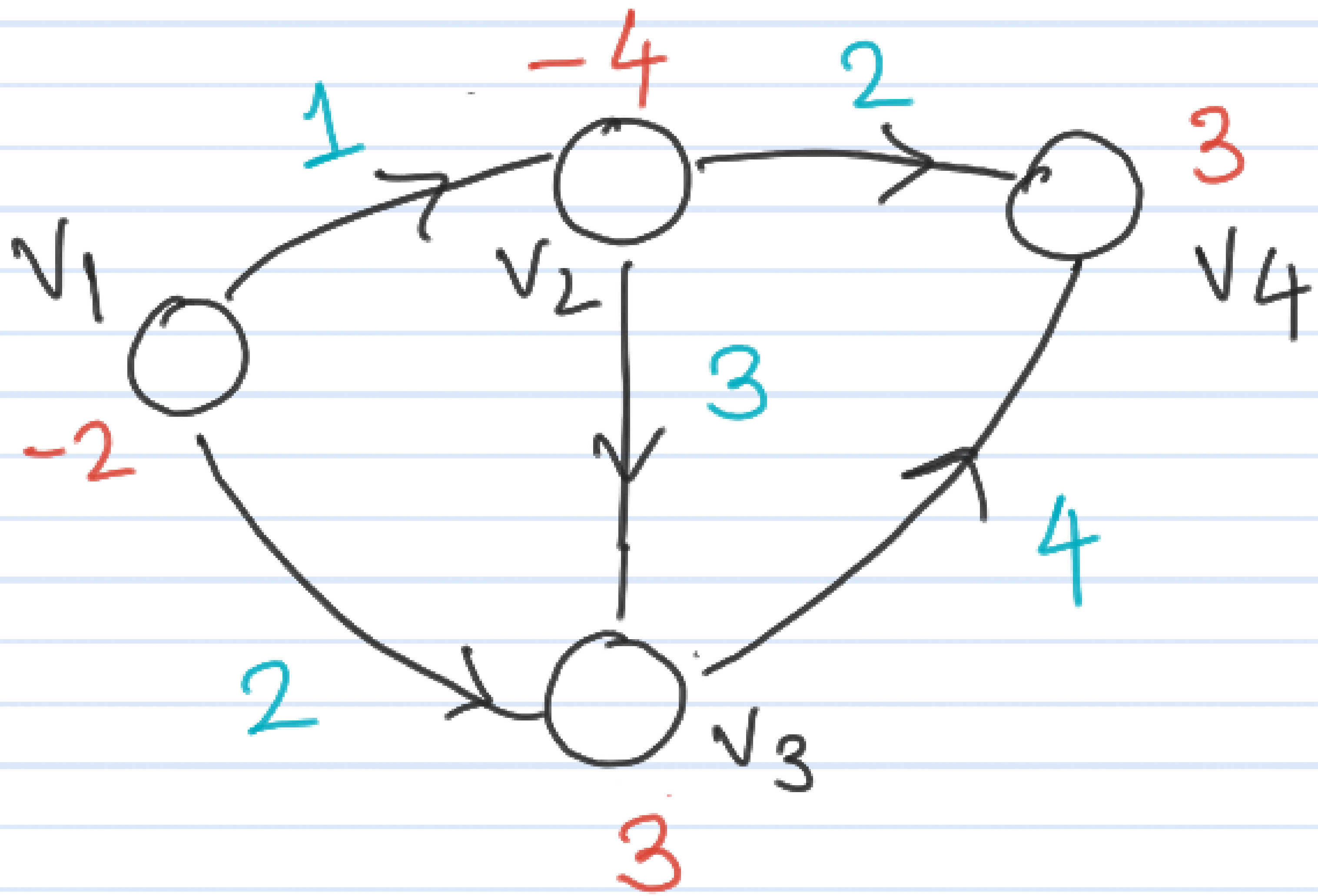
s.t. (1) $f(e) \leq c(e)$ (2) $f_{in}(v) - f_{out}(v) = d(v)$

Demands and Circulation : Example

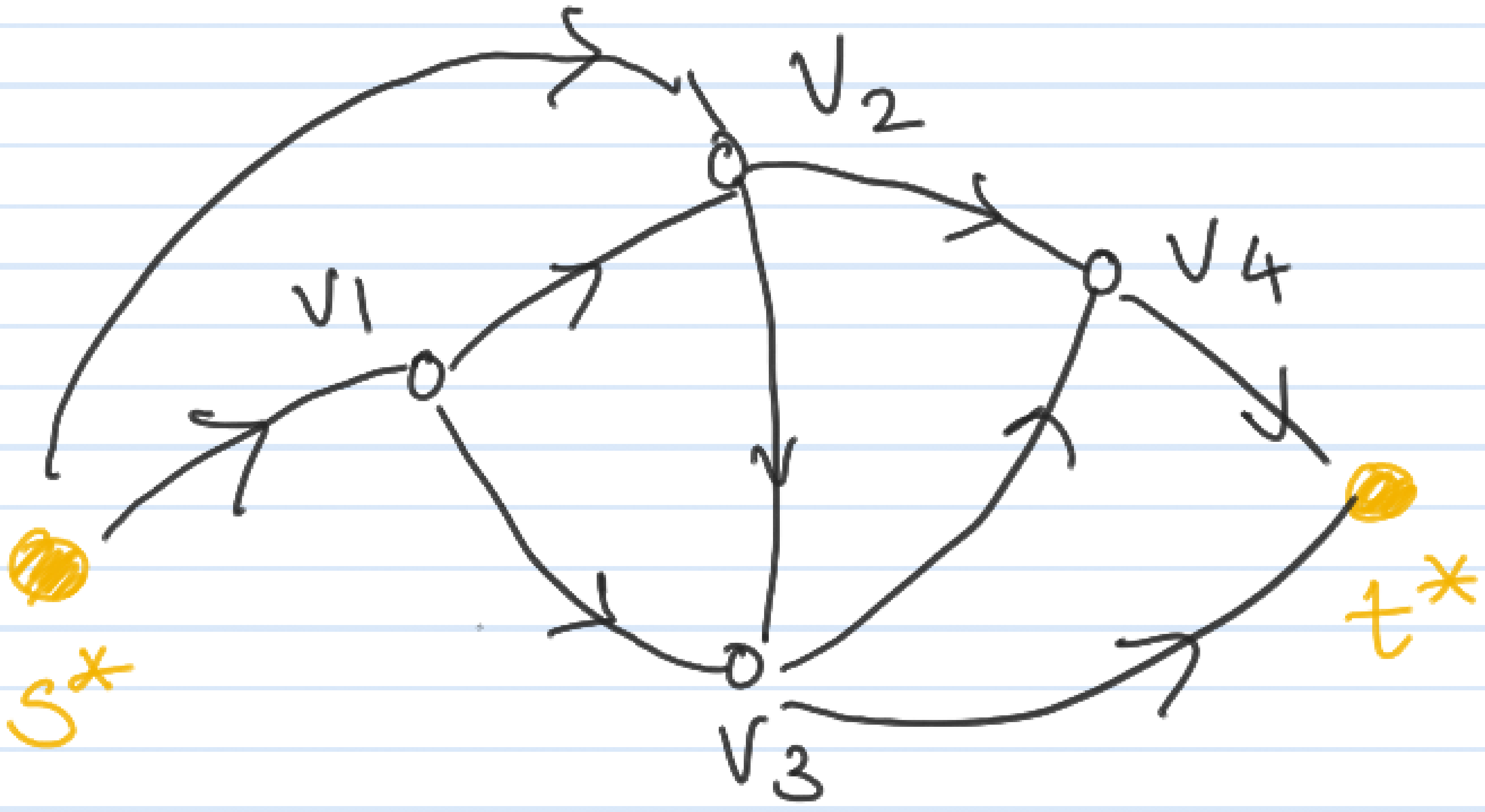


is there a feasible flow?

Demands and Circulation: Reduction

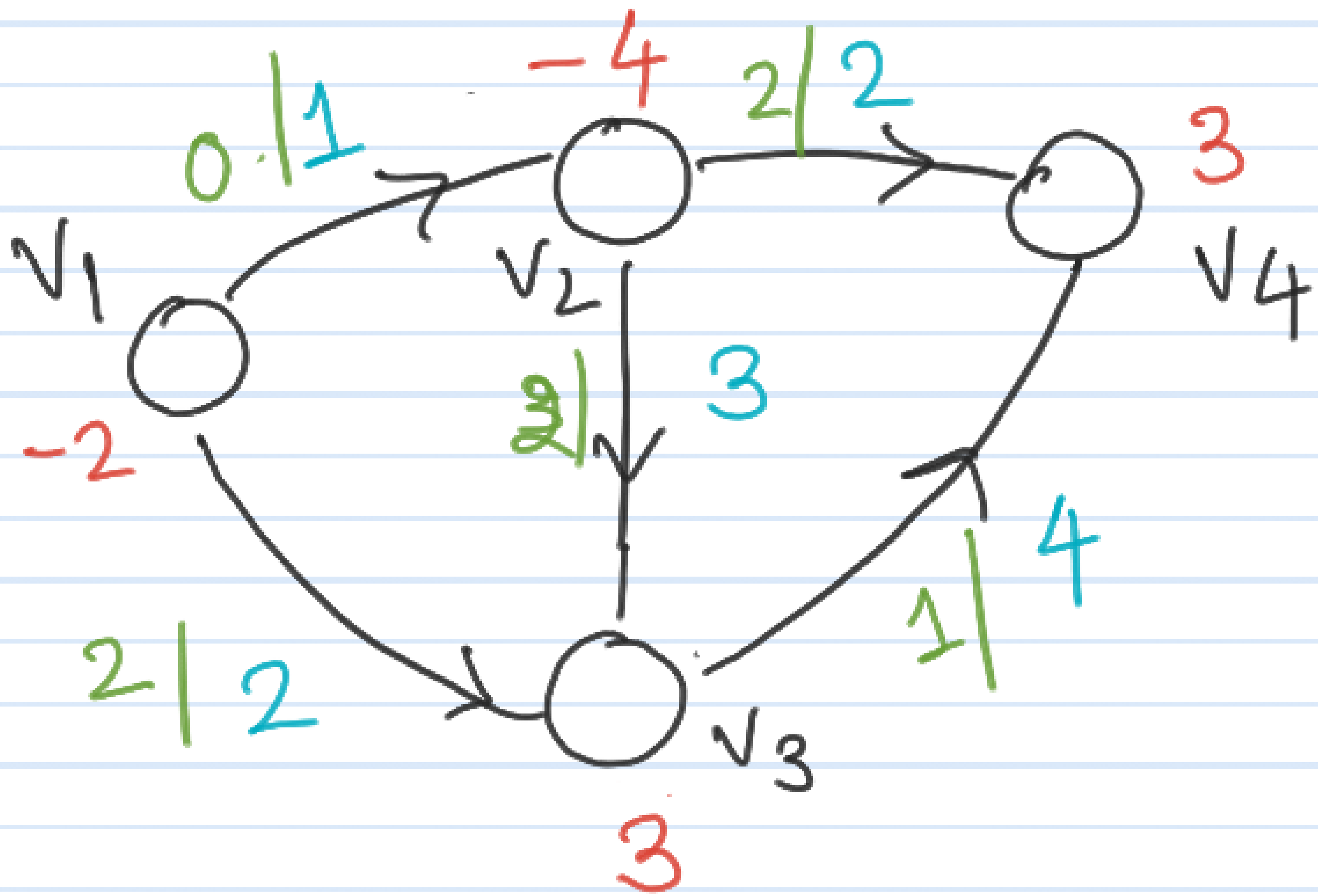


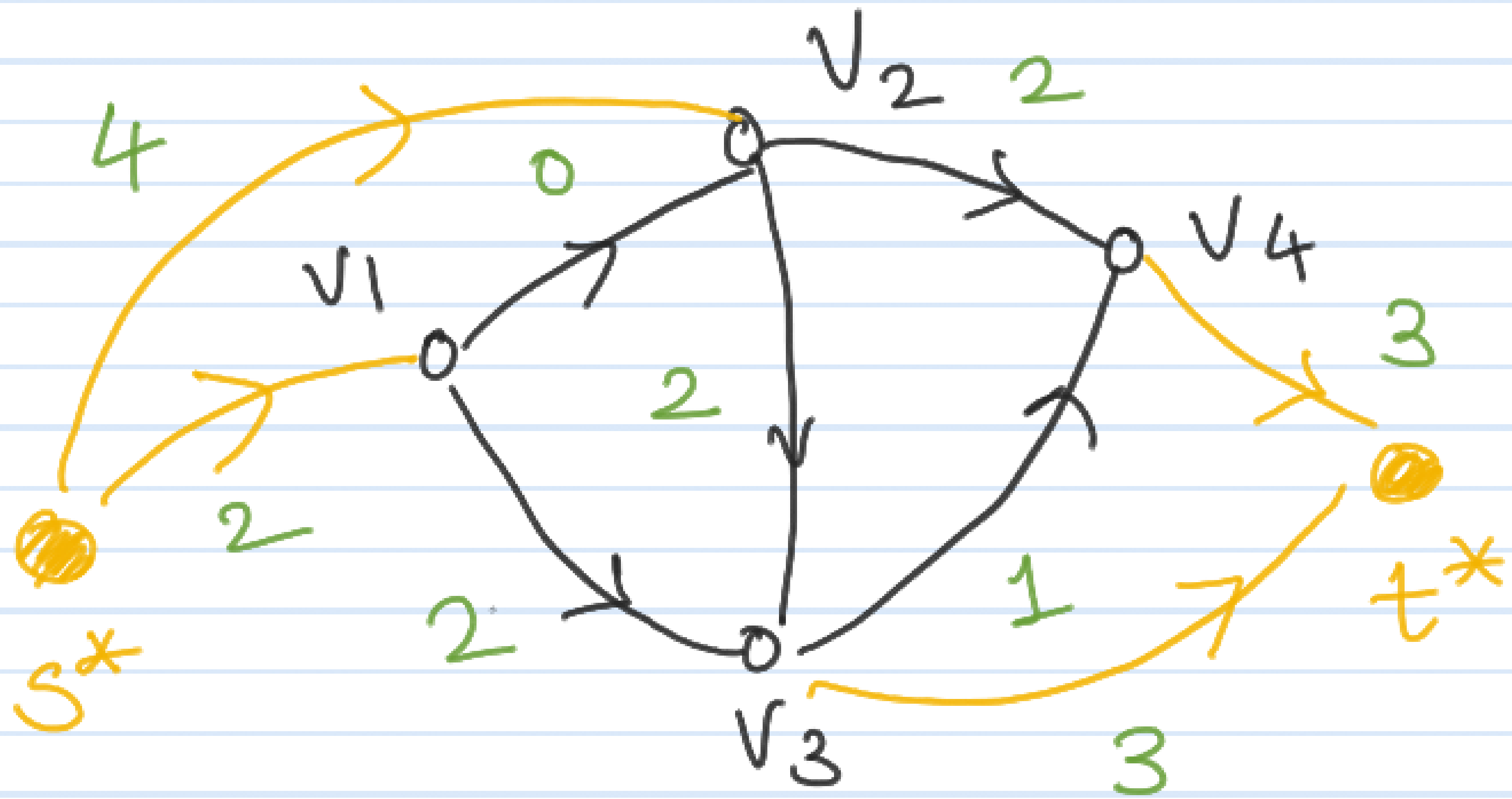
is there a feasible flow?



What is the max flow?

Demands and Circulation : Reduction





is there a feasible flow?

What is the max flow?

Circulations, Demands and lower bounds

Input : $G(V,E)$ $c(e) > 0$; $l(e) \leq c(e)$

$d(v)$ for every vertex

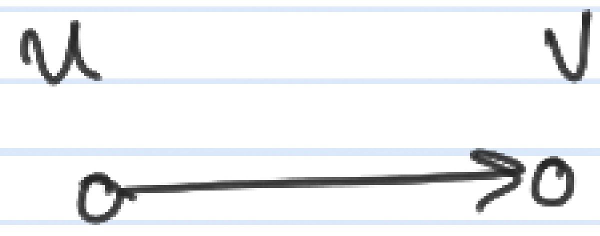
if it exists

Goal : Compute a feasible flow satisfying demands and lower bounds.

Demand and lower bounds $\Rightarrow$ Demands and No lower bounds $\Rightarrow$ Vanilla Maxflow problem

G	H	H'
feasibility yes / no answer	feasibility yes / no	optimization (max).

Demands with lower bounds $\rightarrow$ Demands w/o lower bounds



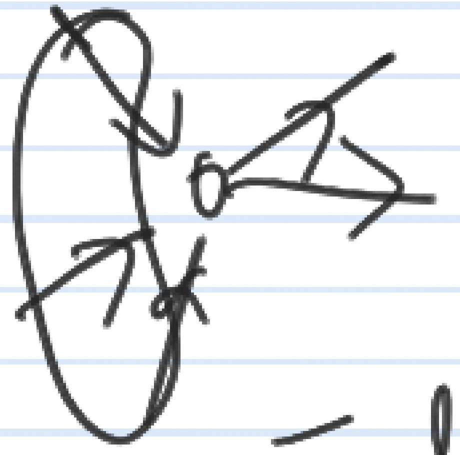
(1) capacity constraint?

$d(v) - L(v)$

(2) Demand?

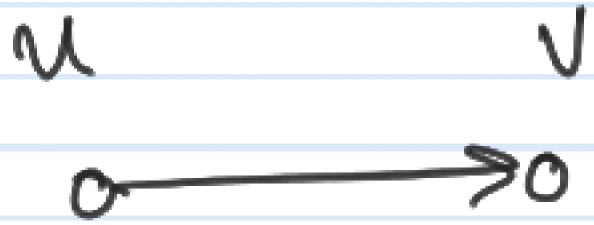
$l(e) \leq c(e)$

Suppose I send $l(e)$ along all edges?



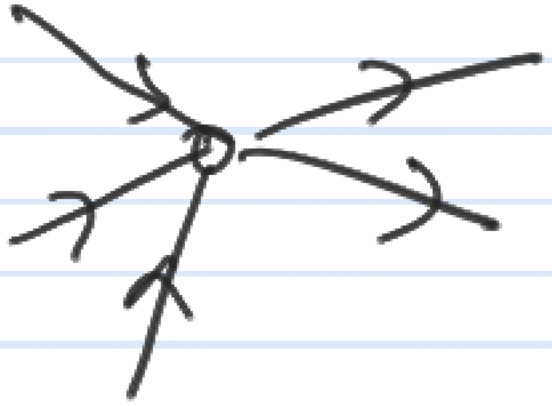
$$L(v) = \left(\sum_{in} l(e) - \sum_{out} l(e) \right) \cancel{\neq d(v)}$$

Demands with lower bounds $\rightarrow$ Demands w/o lower bounds



$l(e) \le c(e)$

Suppose I send $l(e)$ along all edges?



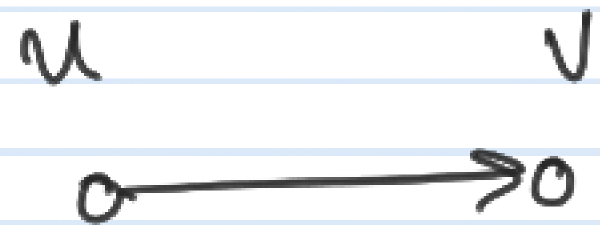
$$\underbrace{\sum_{e:in} l(e) - \sum_{e:out} l(e)}_{L_v}$$

if $L_v = d(v)$ we are done

Demands with lower bounds $\rightarrow$ Demands w/o lower bounds

$$d'(v) = d(v) - L(v)$$

$$c'(e) = c(e) - \ell(e)$$



Note that there is no $\ell'(e)$ in H.

$\ell(e) \le c(e)$

Suppose I send $\ell(e)$ along all edges?

Demands with lower bounds $\rightarrow$ Demands w/o lower bounds

G: $\ell(e) \le c(e)$ $\Rightarrow$ H: $c'(e)$

$d(v)$ for all vertices $\quad$ $d'(v)$ for all vertices

Claim: G admits a feasible circulation iff H admits a feasible circulation.