CS2700 : PROGRAMMING AND DATA STRUCTURES.

BINARY SEARCH TREES

(HEIGHT BALANCED TREES)

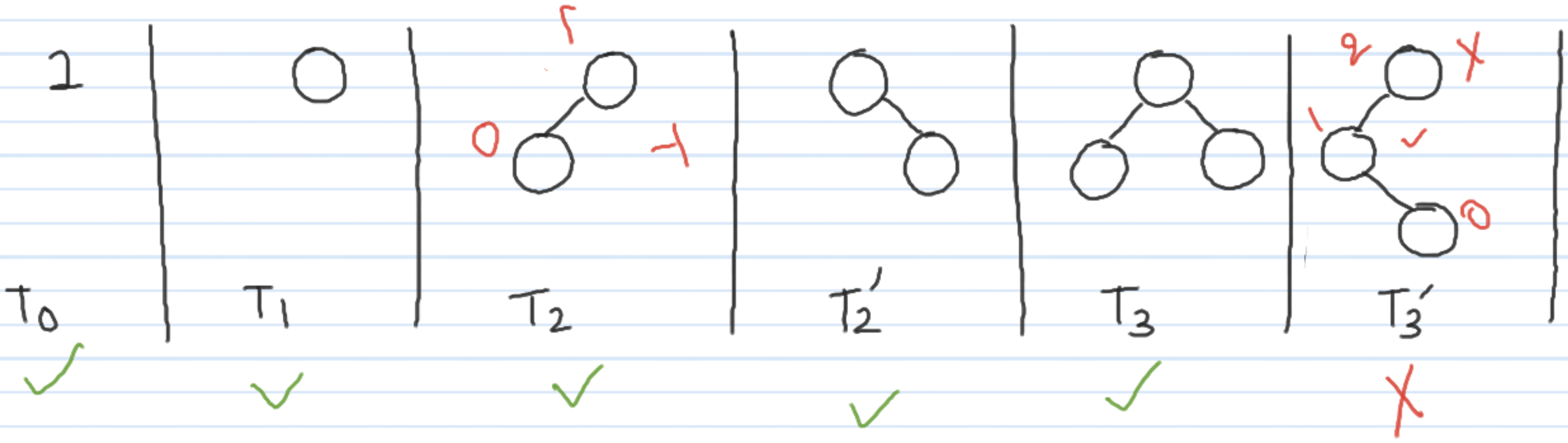
GOALS :

- WHAT IS HEIGHT BALANCE?
- HOW TO ACHIEVE IT?

HEIGHT BALANCED TREES

[AVL TREES]

- AT every node height difference can be at most 1.

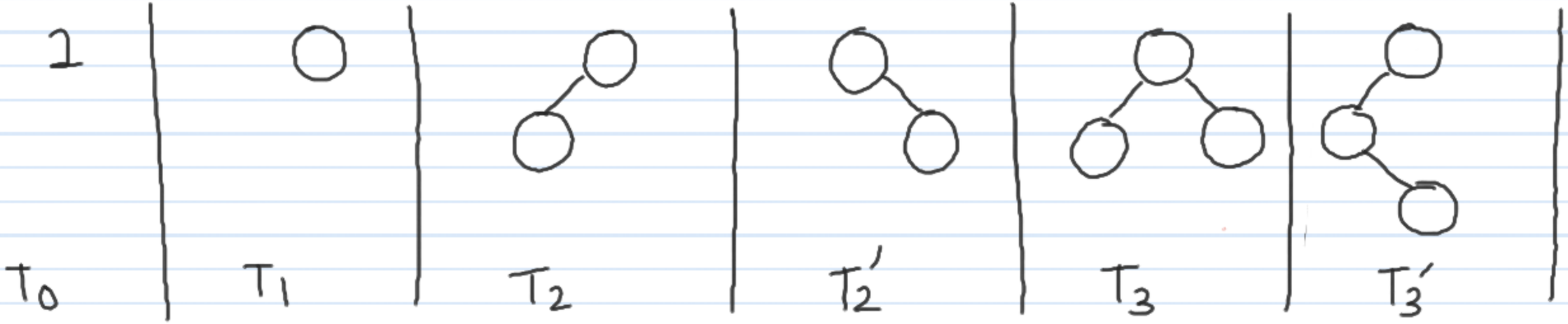


HEIGHT BALANCED TREES

[AVL TREES]

- AT every node height difference can be at most 1.

Adelson Velski Landis



INSERTION INTO AN AVL TREE

Recall

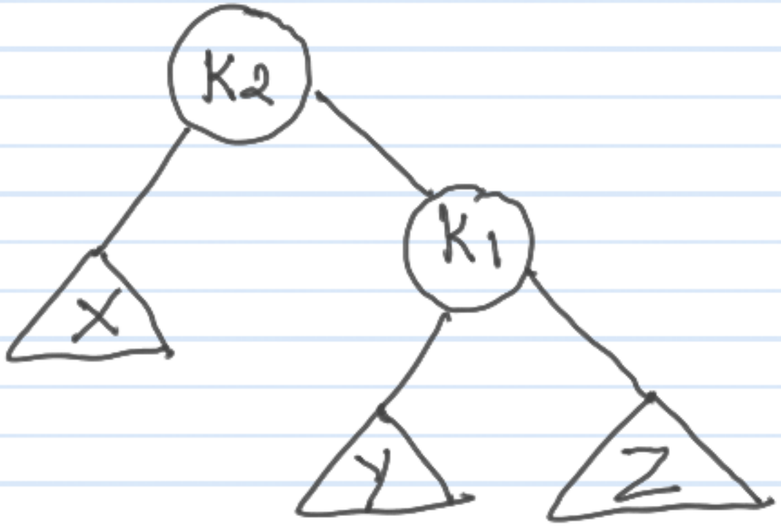
- Height balance definition
- Search property has to be respected.
- α be the deepest node which is imbalanced.

4 cases:

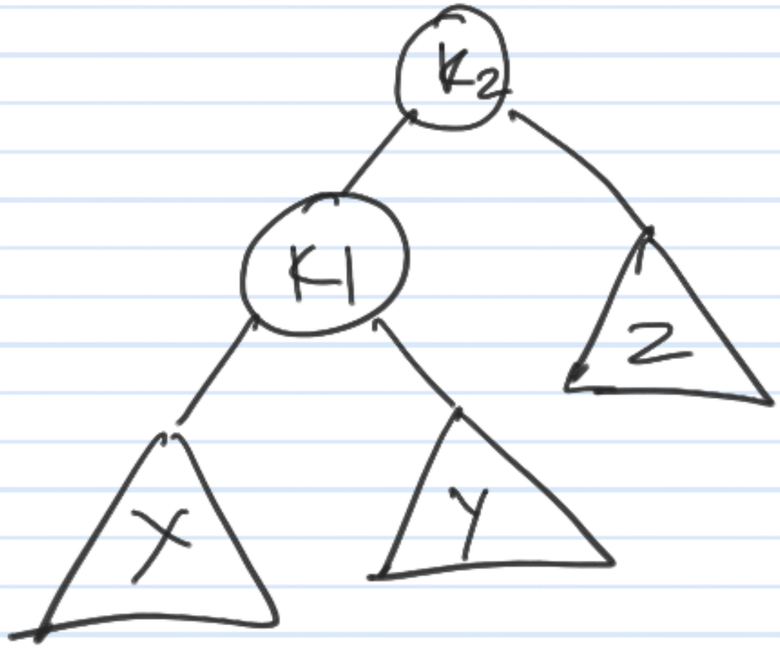
(1) Left left (2) Left Right (3) Right - left (4) Right Right

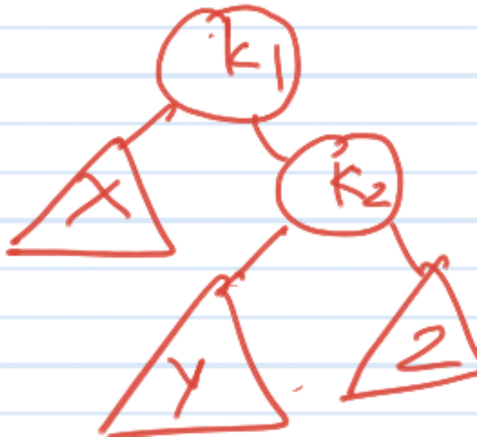
What is a rotation?

Left Rotation

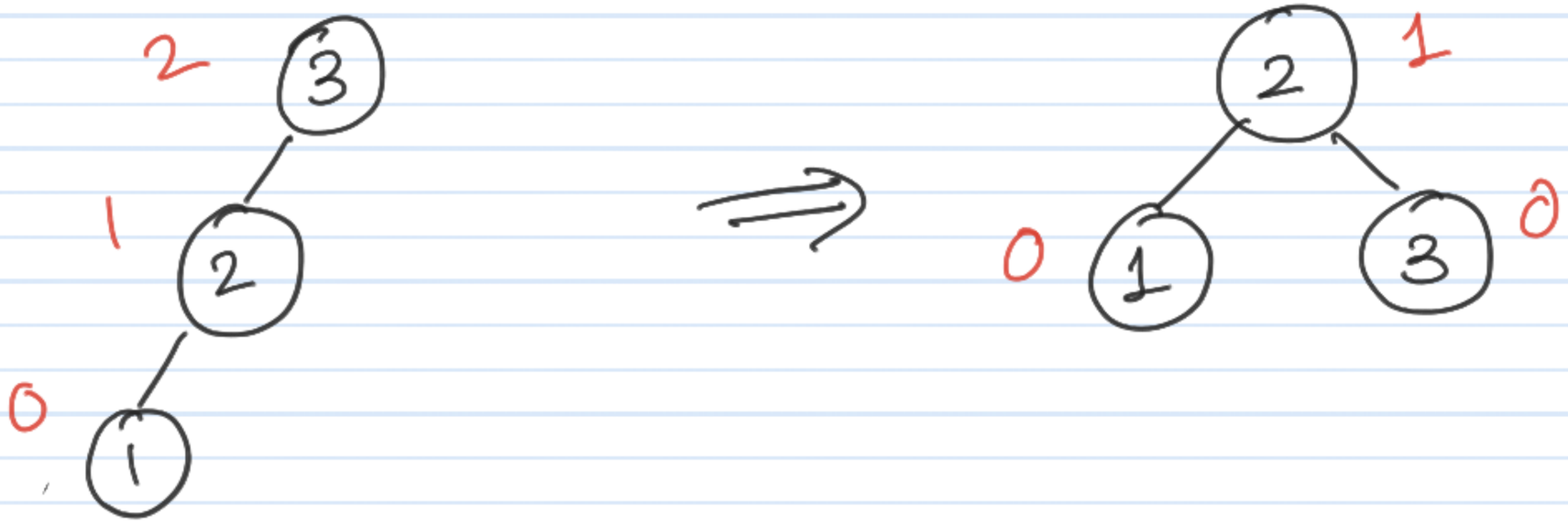


Right Rotation

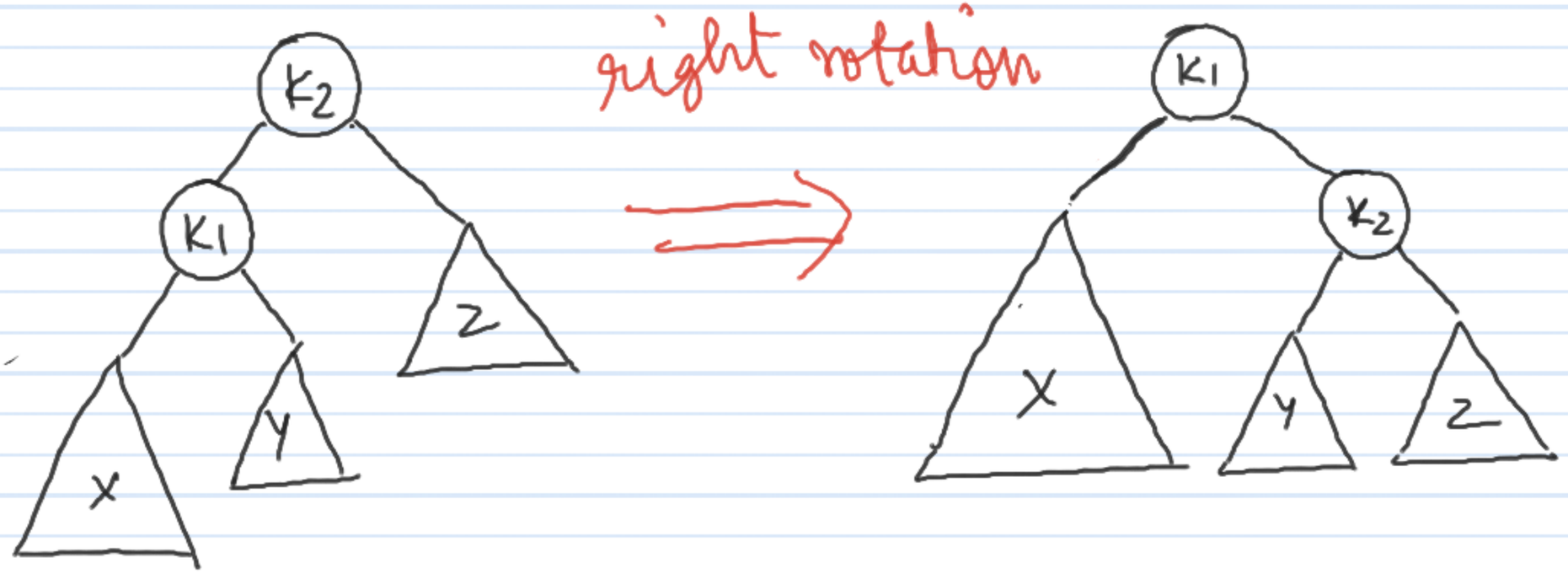




Rotation changes height of nodes.



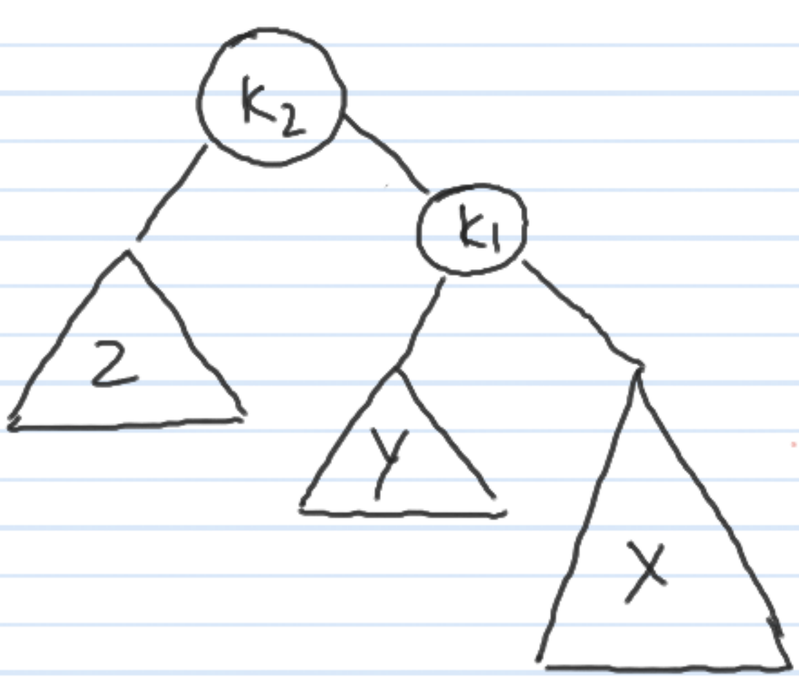
AVL TREE INSERTION : case 1 (left-left)

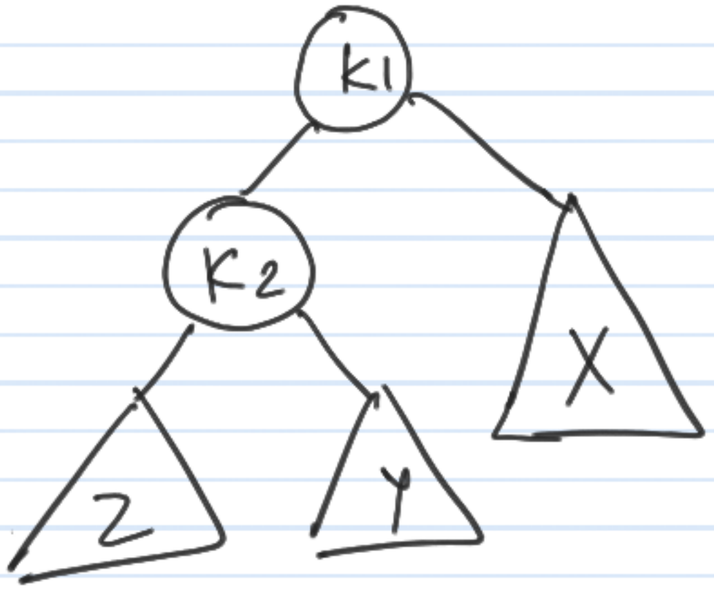


BST order?

- Pull K_1 and rotate it (shake it) and let gravity (bst property) take care.
- X ↑ Y — Z ↓

AVL TREE INSERT : CASE 4 (Right - Right)

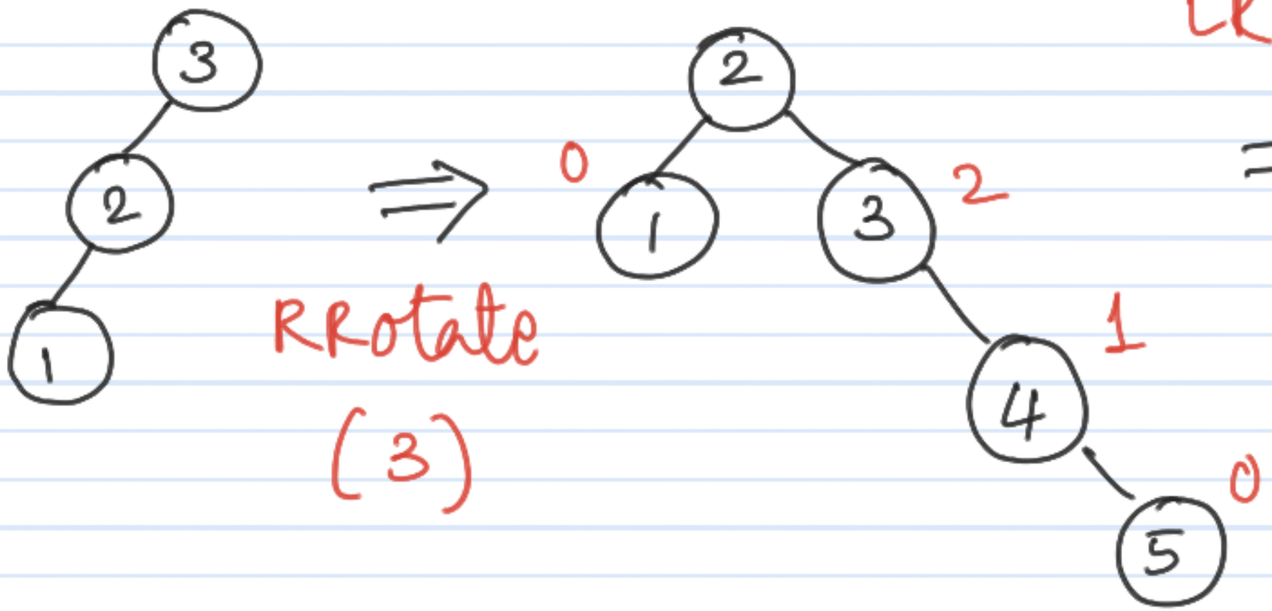


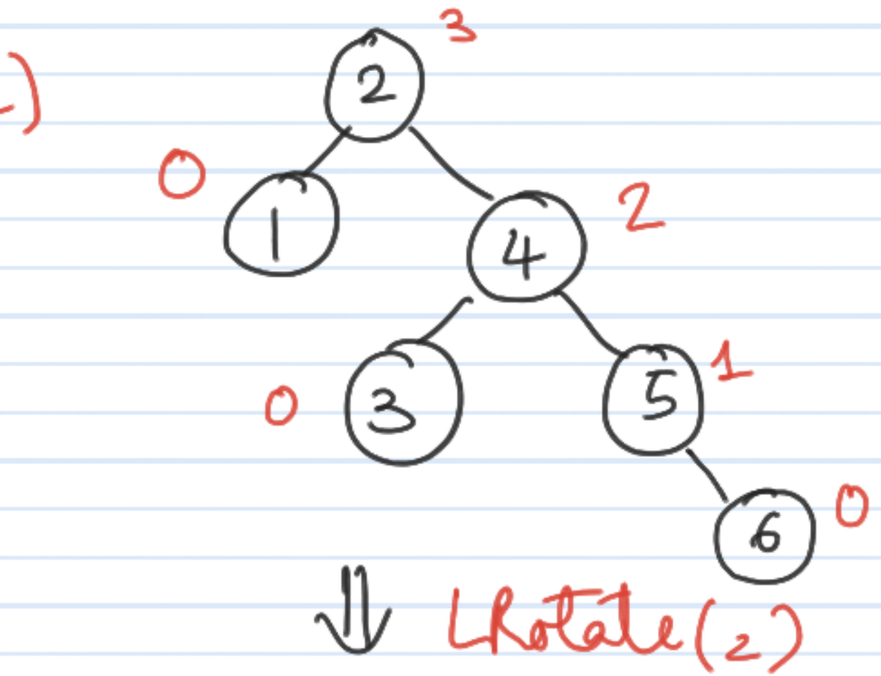


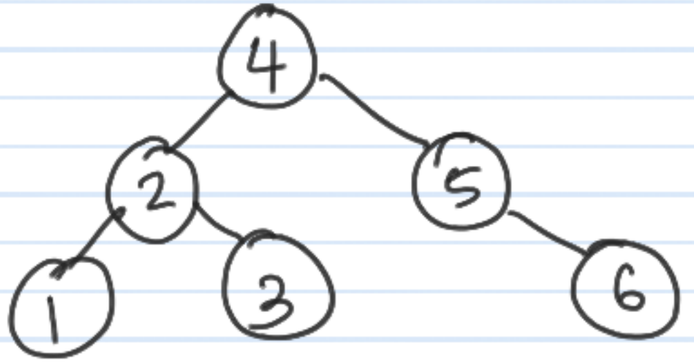
subtree X after insert.

AVL TREE INSERT : EXAMPLE.

insert 3, 2, 1, 4, 5, 6, 7



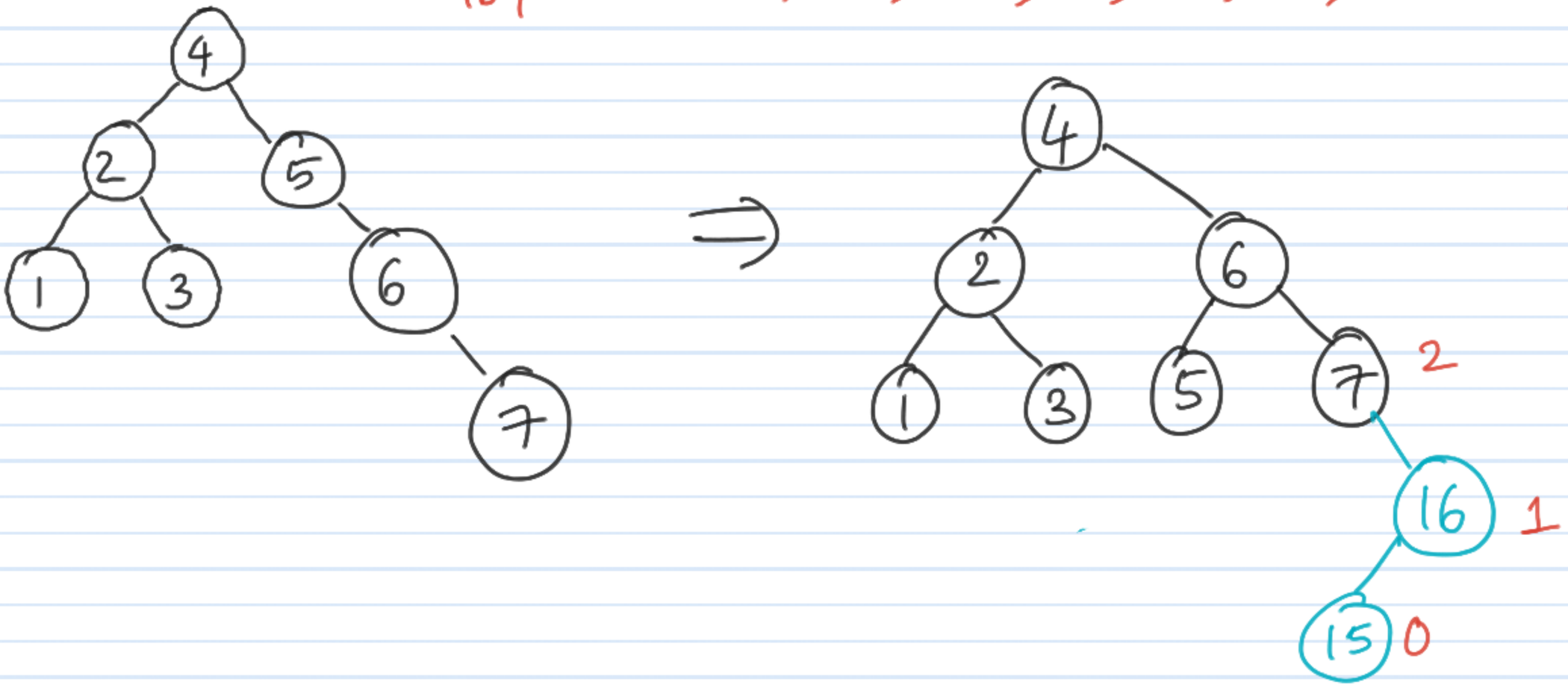




AVL TREE INSERT : EXAMPLE.

insert 3, 2, 1, 4, 5, 6, 7

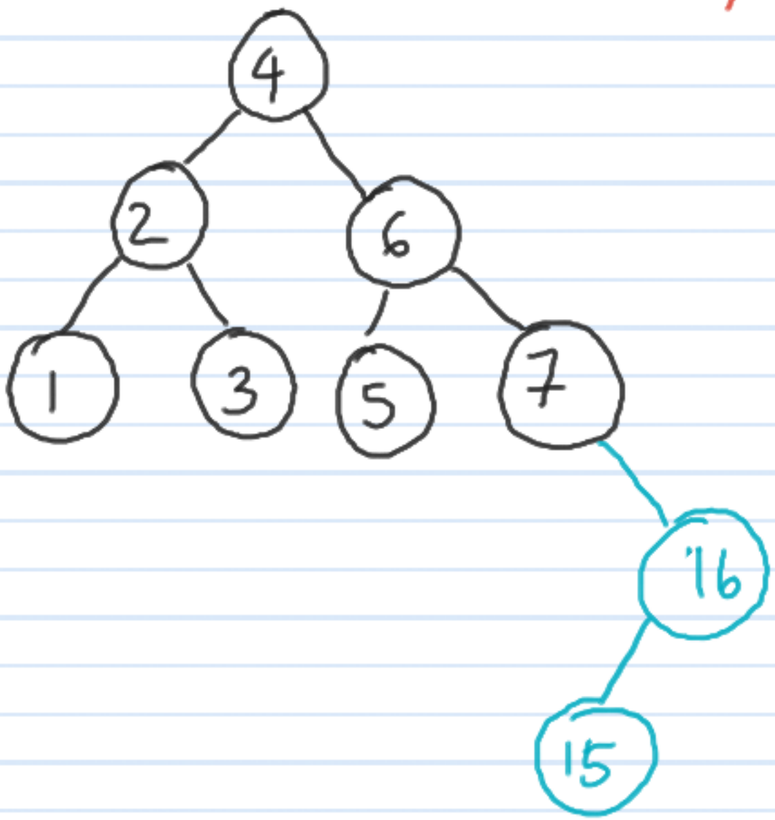
16, 15 14, 13, 12, 11, 10, 9, 8

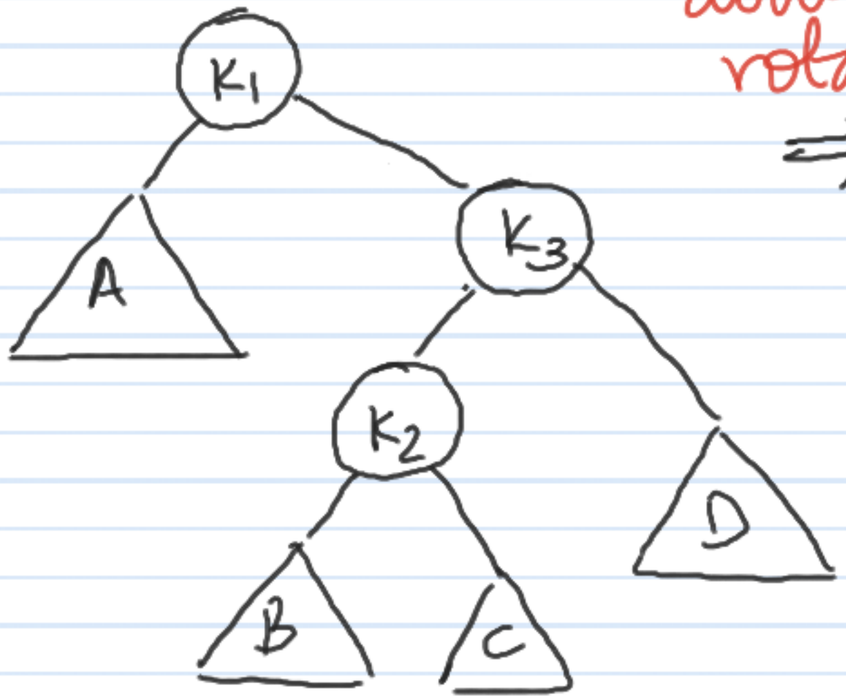


AVL TREE INSERT : EXAMPLE. (Case 3 Right left)

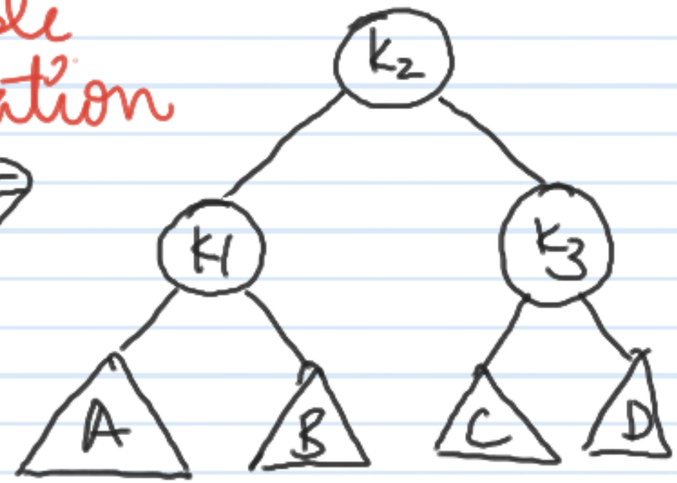
insert 3, 2, 1, 4, 5, 7

16, 15, 14, 13, 12, 11, 10, 9, 8

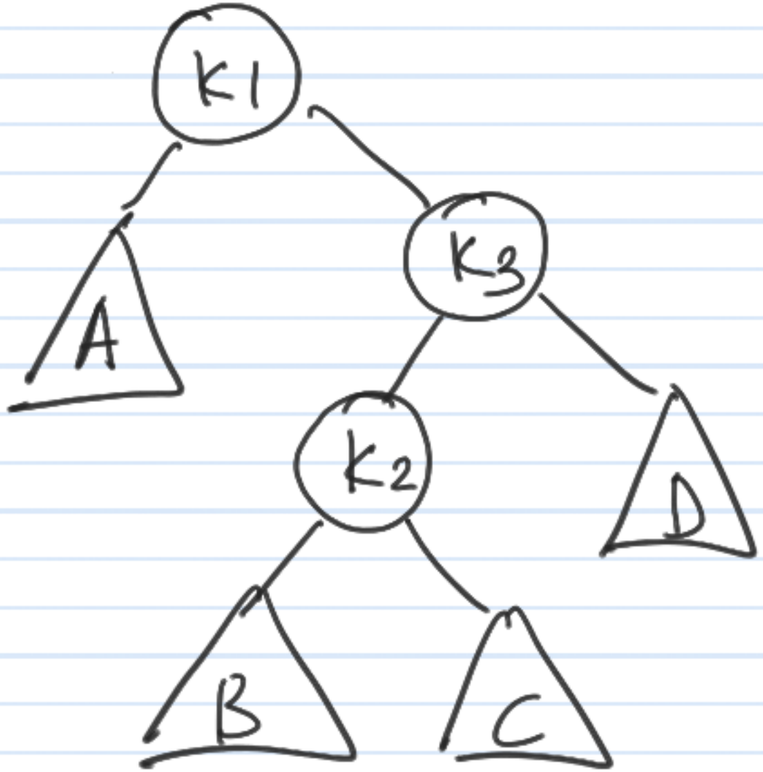




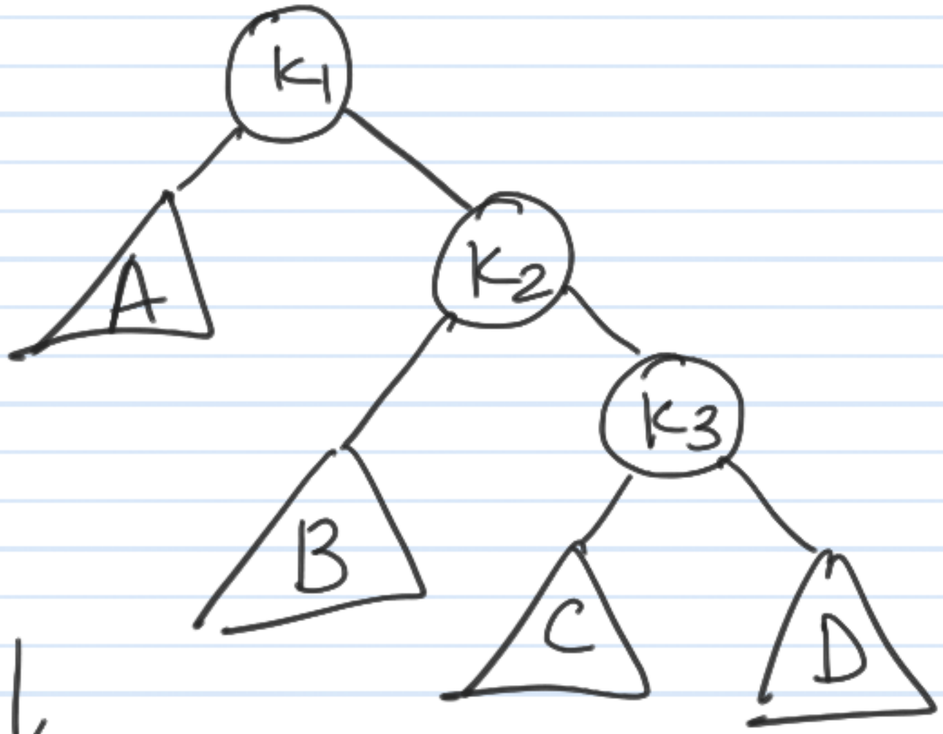
double rotation ⇒



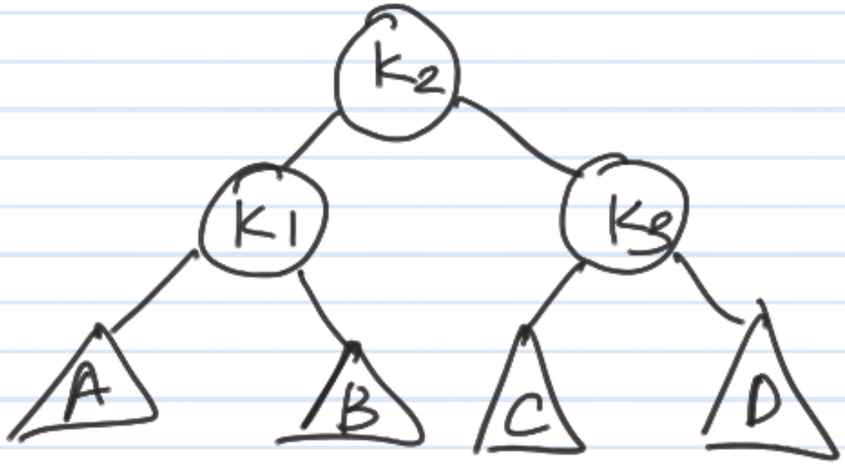
RRotate (k_3)



⇒



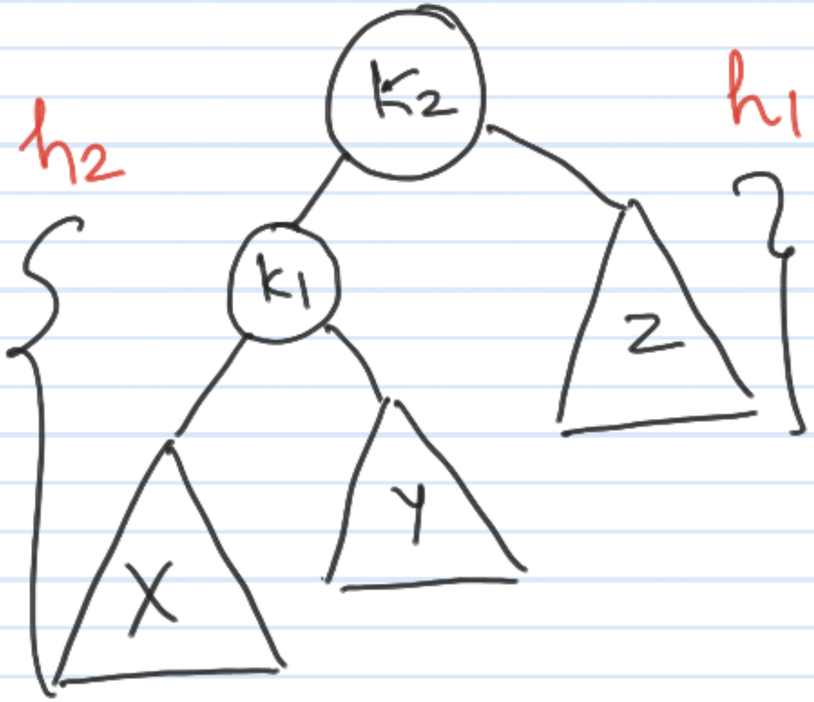
LRotate (k_1) ⇓



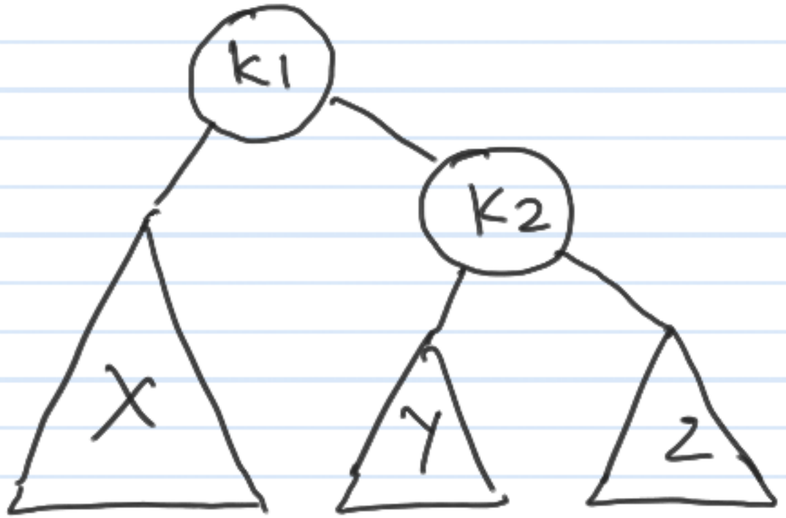
A, k_1, B, k_2, C, k_3, D

Height balance is restored using rotation

(Left Left case 1)



RRotate (k_2)

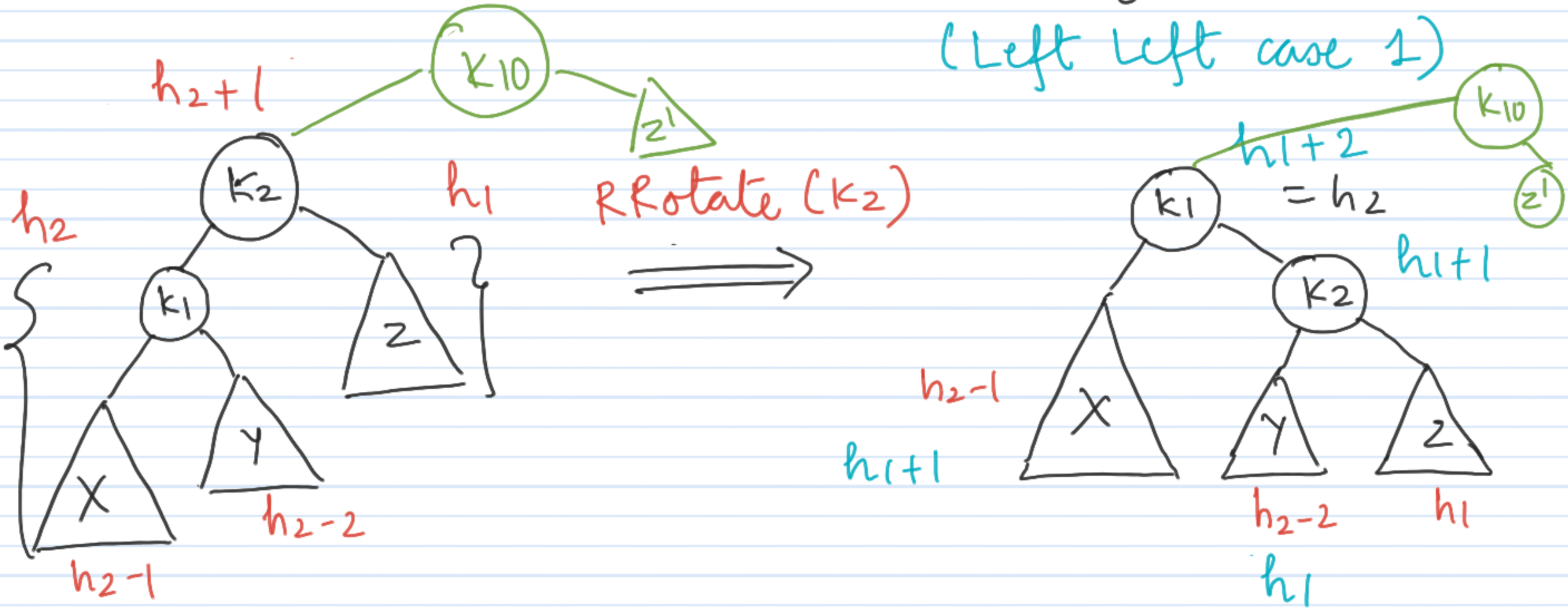


$h_2 > h_1$ Why?

$h_2 - h_1 = \underline{2}$

Height balance is restored using rotation

(Left Left case 1)

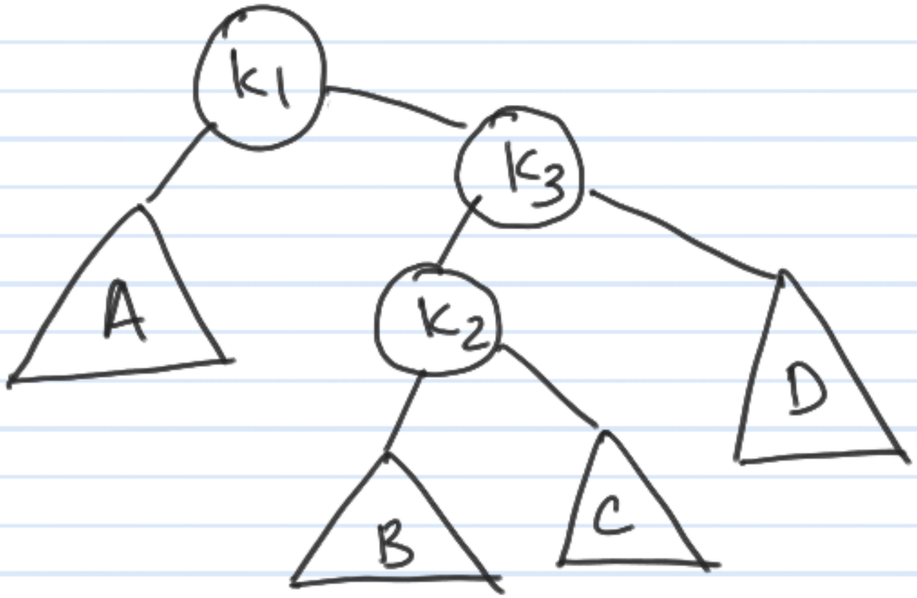


$h_2 > h_1$ Why?

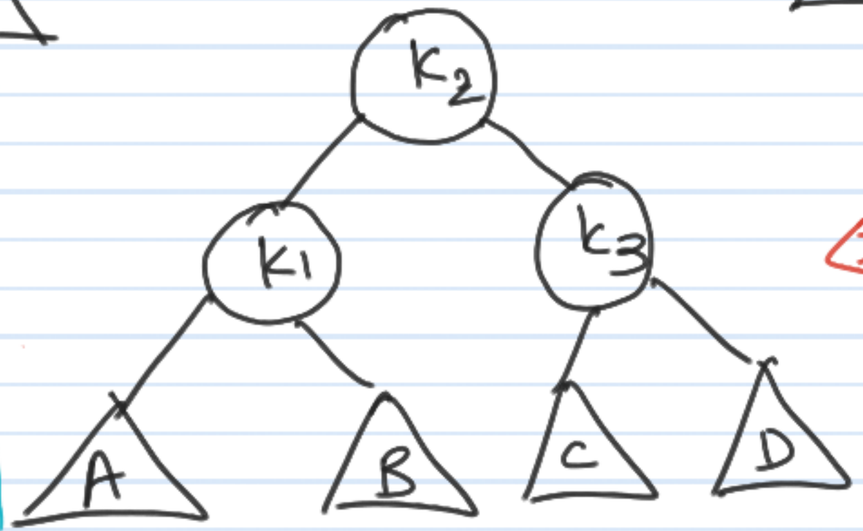
$h_2 - h_1 = 2$

Height balance is restored using rotation

(Right - Left case)



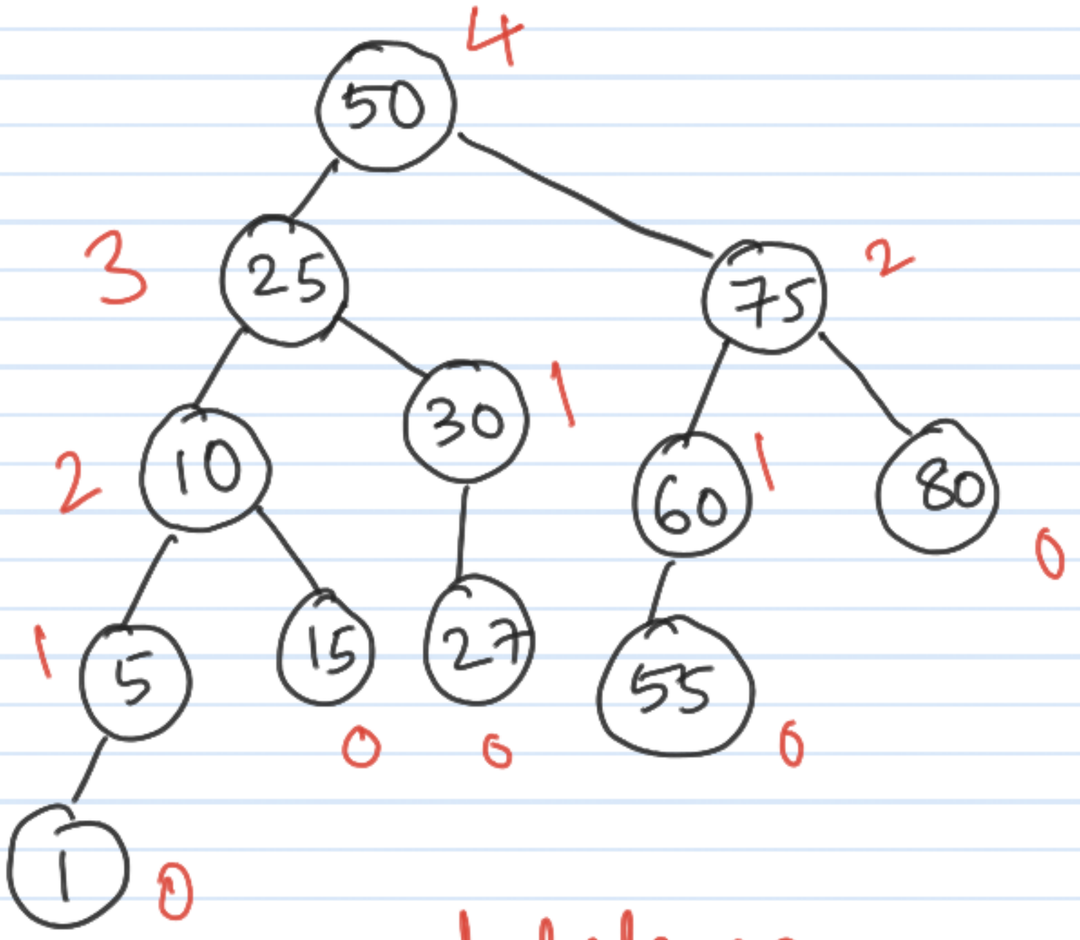
RRotate (k_3)



LRotate (k_1)

Analyse heights for individual rotations.

Deletion in AVL Trees [overview]



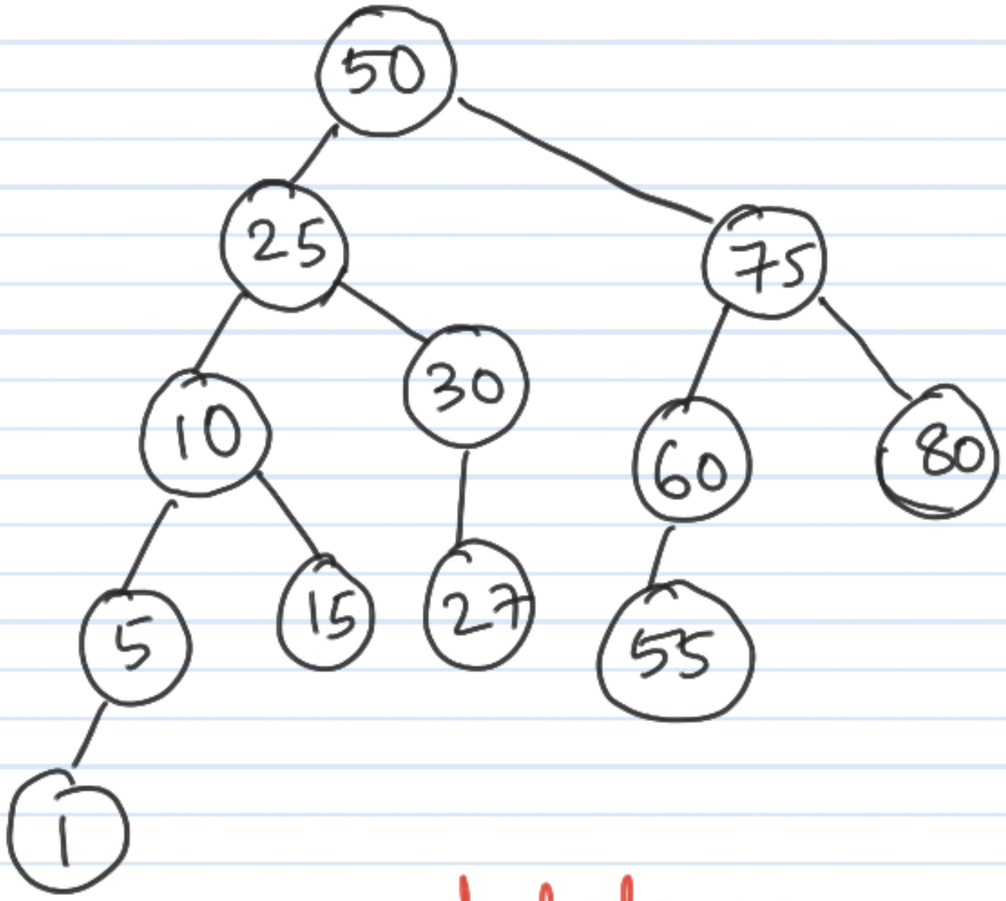
delete 80

- simplest case in BST deletion

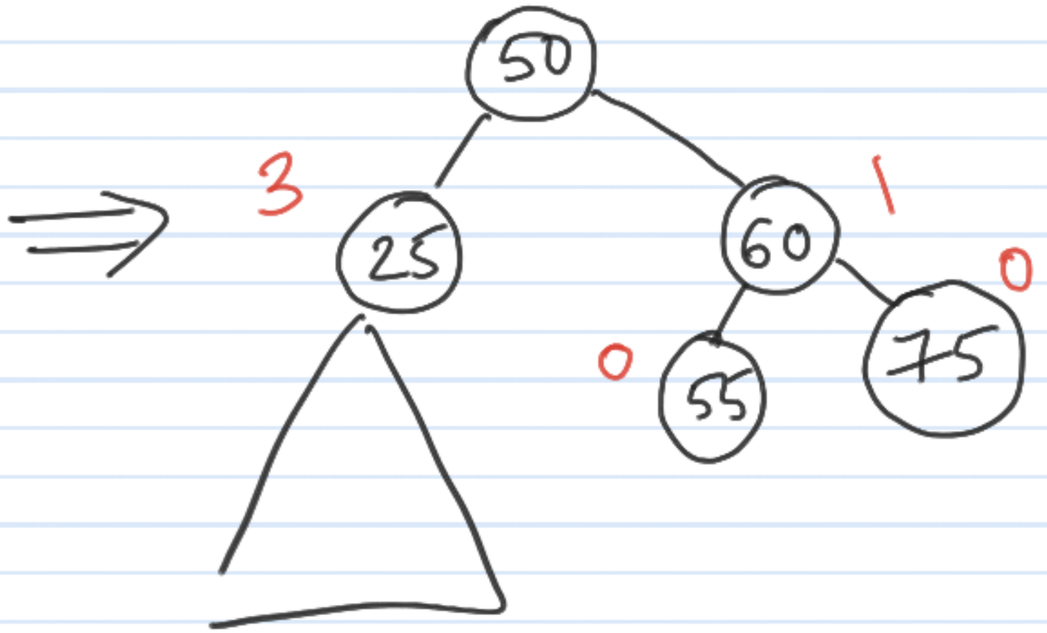
- as earlier convert internal node deletion to a simpler case.

- rotations to restore balance.

Deletion in AVL Trees [overview]



delete 80



- is this tree balanced?