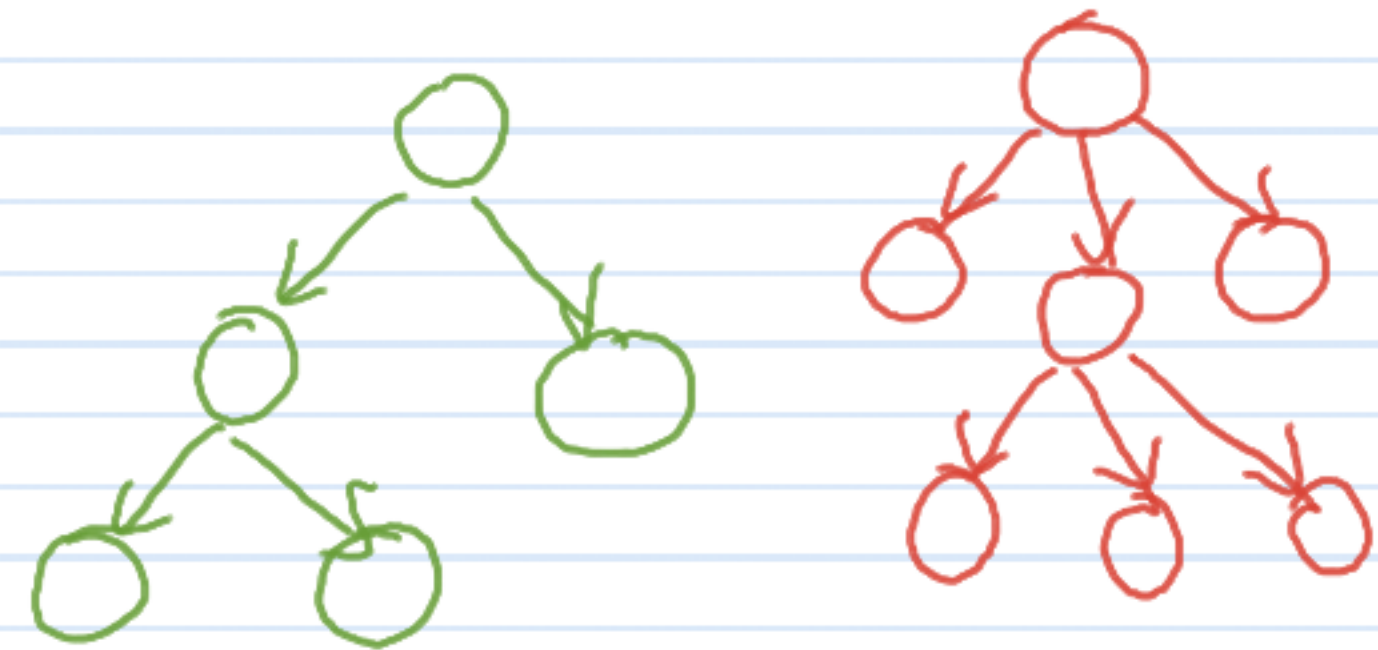


CS 2700 PROGRAMMING AND DATA STRUCTURES

WEEK 7-8 TREES



Why trees?

- Allow storage and access hierarchical data
- Efficient search of data

What is a tree?

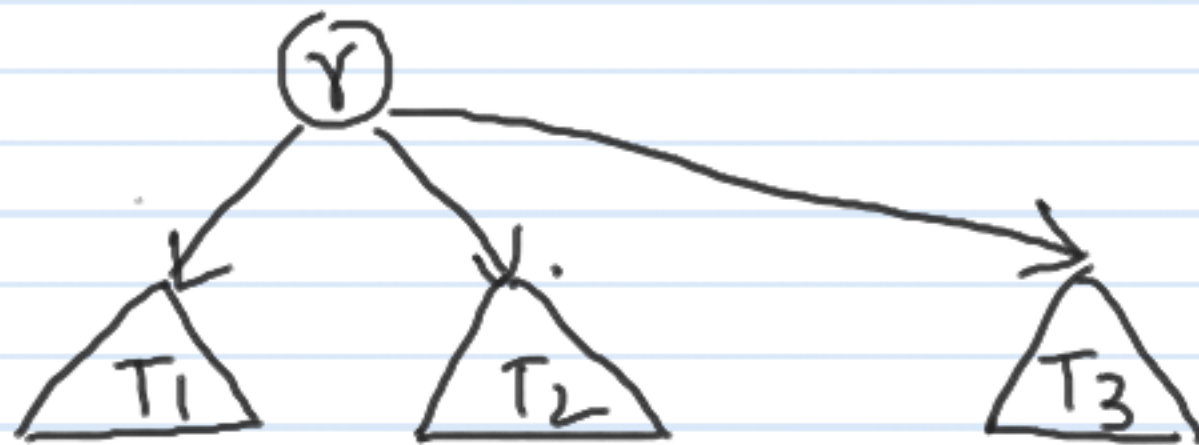
- A **TREE** is a collection of nodes

ELSE :

- collection can be empty (base case)

- A **TREE** consists of a distinguished node r

and zero or more non empty subtrees
 T_1, T_2, \dots, T_k connected by directed edges.

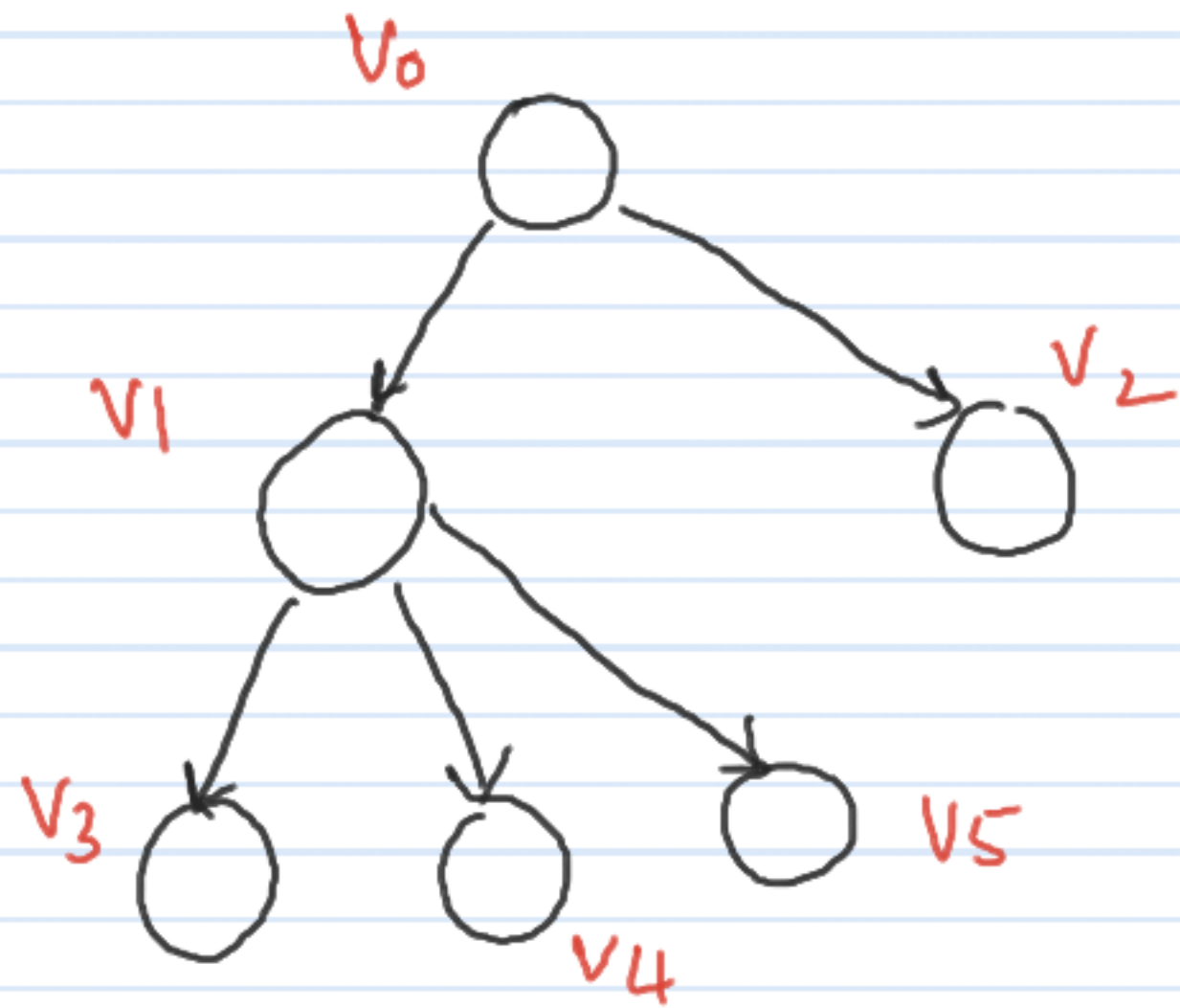


Every node except the root has exactly one parent

Another way to define a tree

- Collection of nodes and directed edges
s.t every node except root has exactly
one parent
- Unique node w/o parent is called **root**
- Nodes without children are called **leaves**
- Rest of the nodes are **internal nodes**

SOME MORE DEFINITIONS



PARENT

SIBLING

GRAND PARENT

ANCESTOR

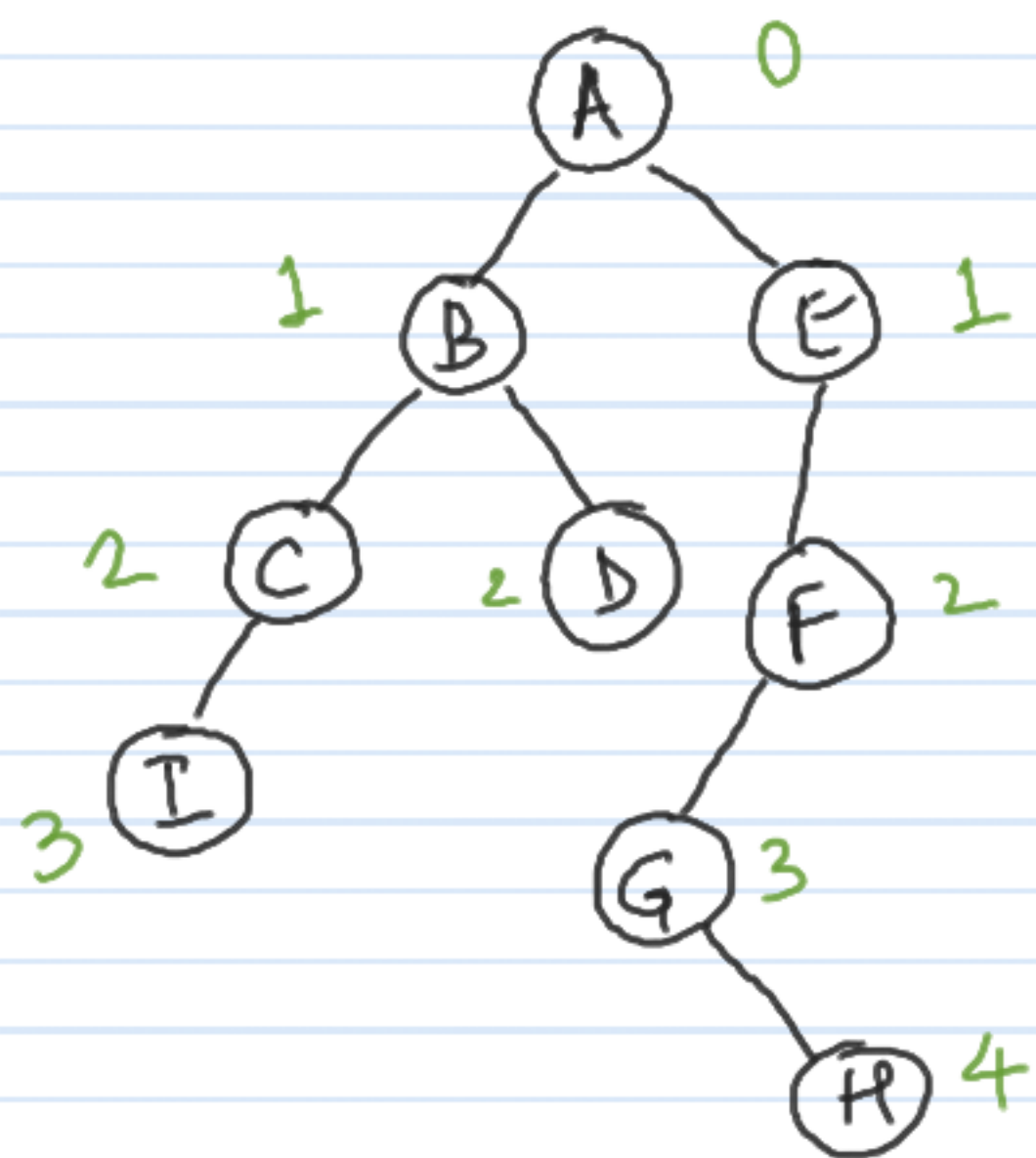
• PATH

• DEPTH

GOALS IN THIS PART

- TREE ADT
- APPLICATIONS WHERE TREES ARE USEFUL
- TREE TRAVERSALS
- SPECIAL TREES (BINARY SEARCH TREES)

BINARY TREES



N nodes

($N=9$) in example

Depth of tree

= depth of deepest leaf

Height of tree:
height of root

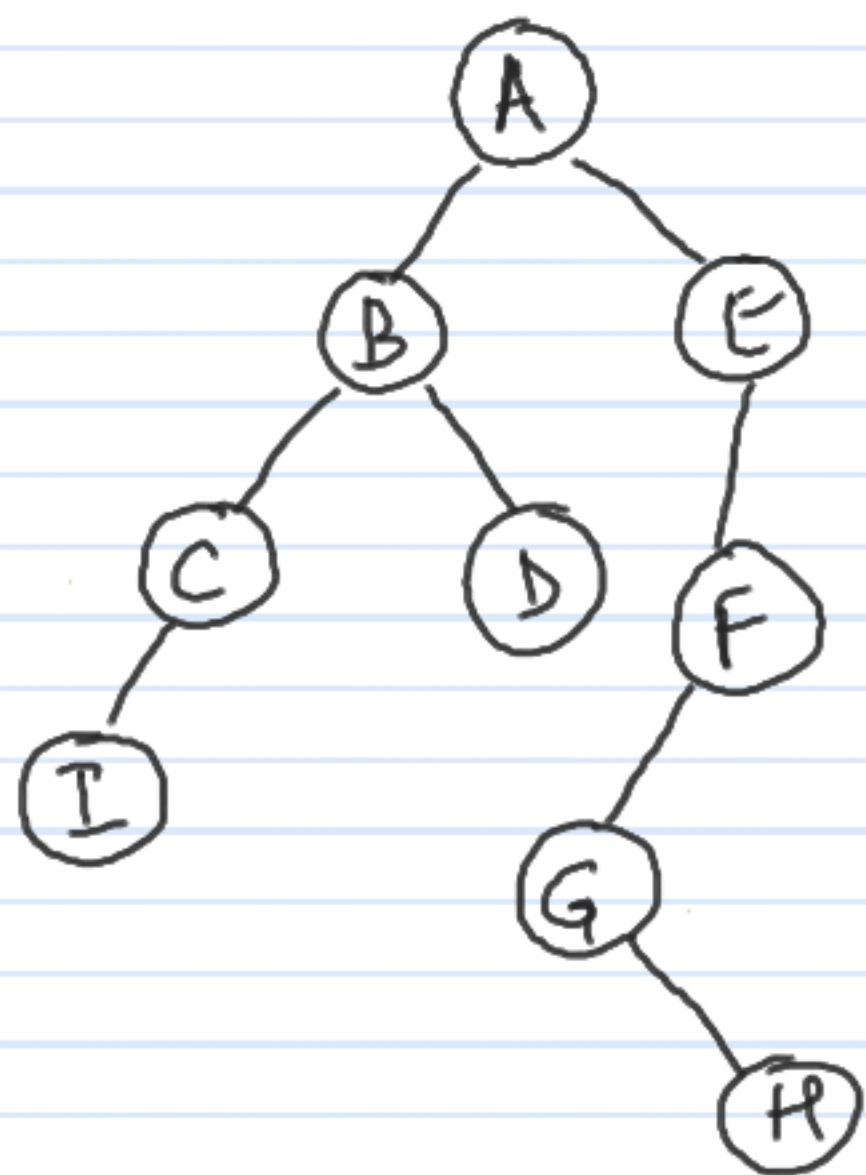
Recall: depth of node ==

length of unique path

from root to node

Height of a node: length of longest path from node to a leaf

BINARY TREES



N nodes

(N=9) in example

max/min height :

??

max/min leaves :

??

of NULL pointers :

$n+1$;

height and size defined recursively

$height(r) = 1 + \max(height(r \rightarrow left)$

A B C I D E F G H

$height(r \rightarrow right))$

BINARY TREES : pre order traversal

```
struct TreeNode {  
    DataType data;  
    struct TreeNode * left;  
    struct TreeNode * right;  
}
```

```
struct TreeNode * root;
```

```
preorder (struct TreeNode r)  
{  
    if (r != NULL)  
        print (r -> data);  
        preorder (r -> left);  
        preorder (r -> right);  
}
```

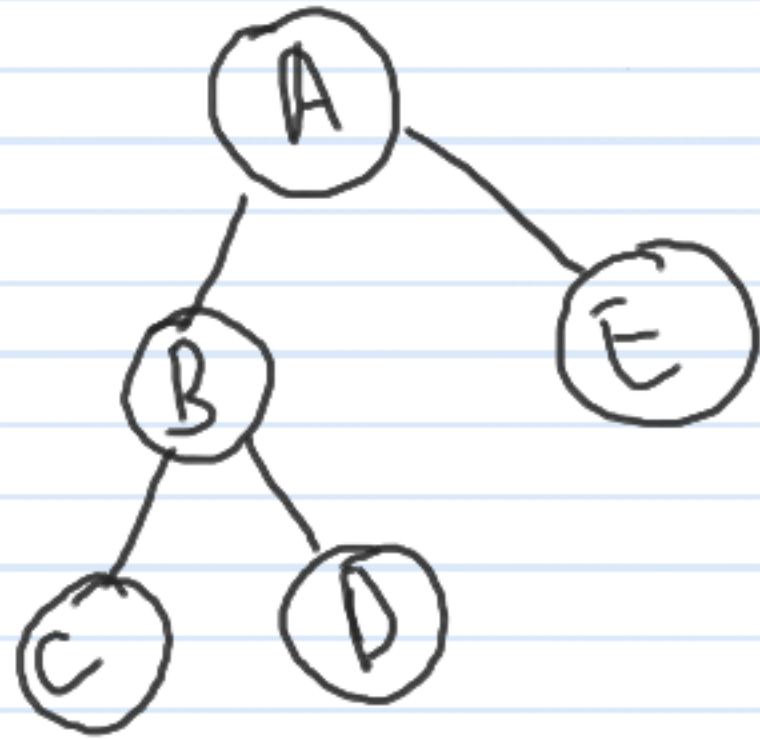

BINARY TREES : PREORDER TRAVERSAL (w/o recursion)

```
struct TreeNode {  
    DataType data;  
    struct TreeNode * left;  
    struct TreeNode * right;  
}
```

```
struct TreeNode * root;
```

```
preorder (struct TreeNode r)  
{  
    stack < struct TreeNode > s  
    if (r != NULL)  
        s.push(r);  
    while (s.empty() != true)  
    {  
        temp = s.top(); s.pop();  
        print(temp->data);  
        s.push(temp->left);  
        s.push(temp->right);  
    }  
}
```

BINARY TREES : PREORDER TRAVERSAL (w/o recursion)



A B C D E

A E B D C



```
preorder (struct TreeNode r)
{
    stack < struct TreeNode > s
    if (r != NULL)
        s.push(r);
    while (s.empty() != true)
    {
        temp = s.top(); s.pop();
        print(temp->data);
        s.push(temp->left);
        s.push(temp->right);
    }
}
```

BINARY TREES : APPLICATIONS

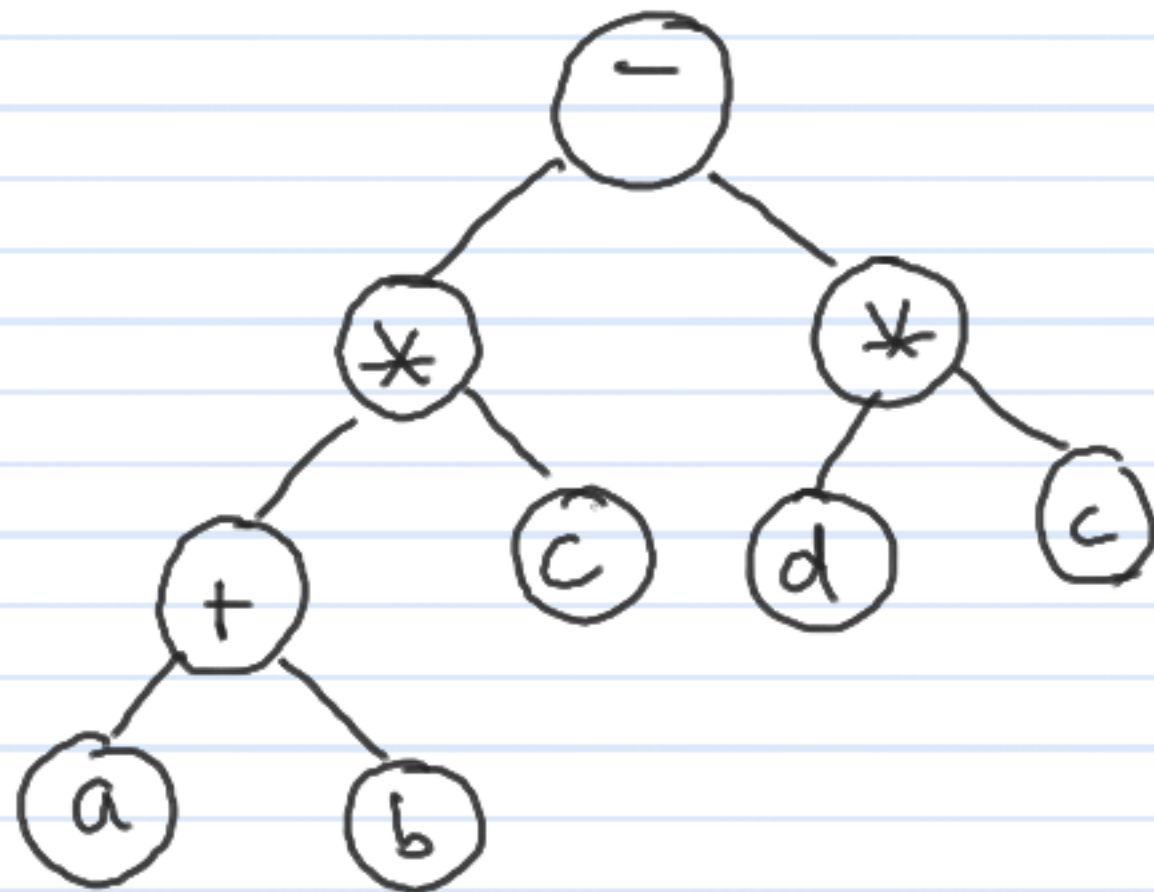
(1) EXPRESSION TREES

(2) CODING (not programming!)

EXPRESSION TREES:

A convenient way to represent expressions

$$(a + b) * c - (d * c)$$

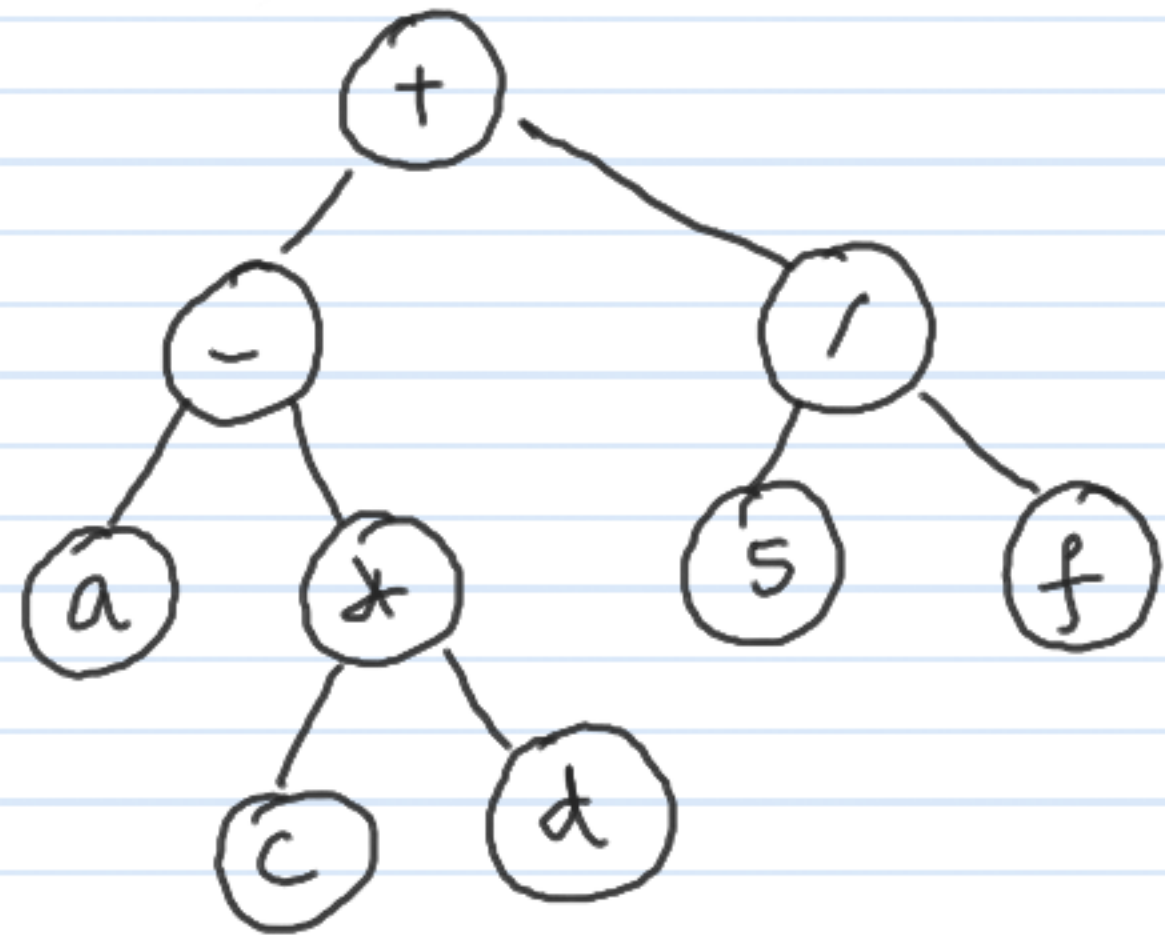


internal nodes;

leaves;

No parenthesis needed.

EXPRESSION TREES



POST ORDER TRAVERSAL OF TREE

LRN

$$(a - (c * d)) + (5 / f)$$

$$a c d * - 5 f / +$$

POSTFIX FORM \rightarrow EXPRESSION TREE CONSTRUCTION.

CODING (not programming!)

Generate codes to transmit data

Like Linked lists trees are also recursive data structures

L	e	26	lower case	46 symbols
i	d	+		
k	:	26	upper case	
e	:	+		
n		10	punctuations	6 bit code is sufficient

CODING (not programming!)

Generate codes to transmit data

Fixed length encoding (say 5 bits)

L 00000

a 00001

b 00010

c 00011

A string of the form is interpreted

00001 010101 10001 00101 00000 10101

uniquely

CODING (not programming!)

Generate codes to transmit data

Fixed length encoding:

- simple and easy decoding
- n char, k length of code for each char

$n \times k$; size of total code

↳ can this be improved?

CODING (not programming!)

Generate codes to transmit data

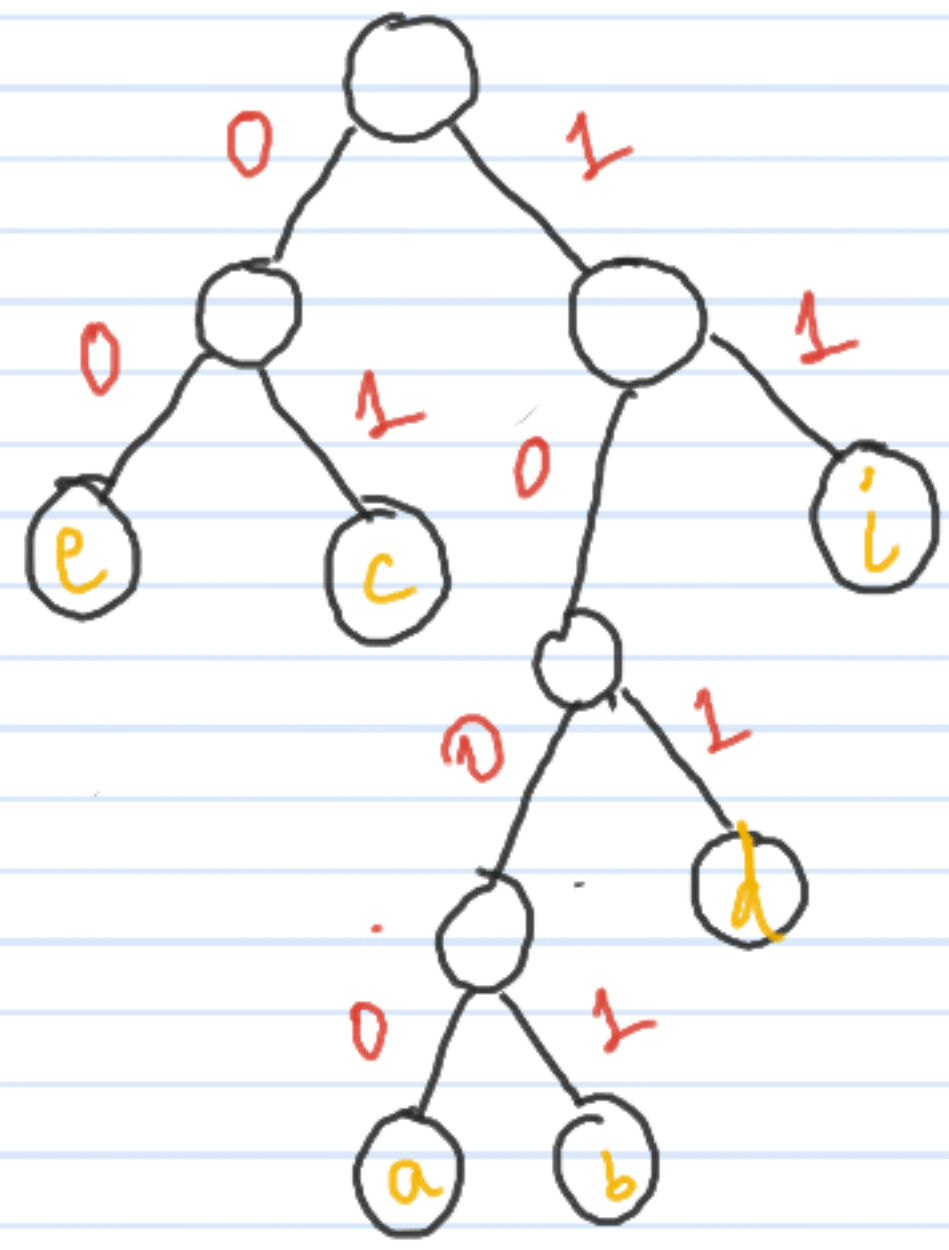
Char	Freq
a	9
b	10
c	15
d	4
e	25
i	30

Reasonable to have shorter codes for frequently occurring characters.

are there such codes?
how are they related to trees?
How to decode?

CODING (not programming!)

Generate codes to transmit data



a	1000
b	1001
c	01
d	101
e	00
i	11

1001 11101
b i d

PREFIX CODES:

no word in code is prefix of any other word in the code

Fixed length codes: always satisfy this property

Variable length codes: may or may not

ex:

a : 00
b : 001 } not prefix

a : 01
b : 001 } ✓ prefix