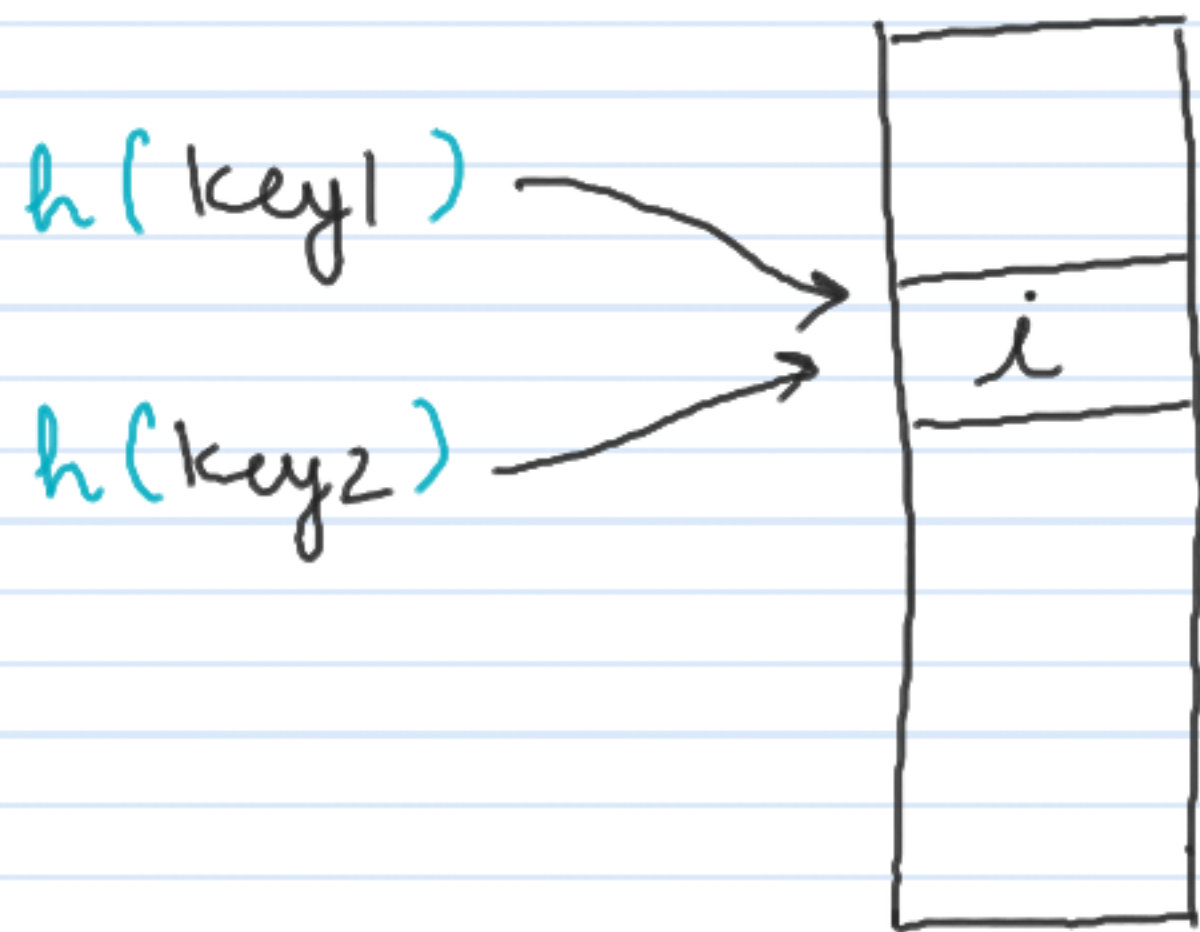


CS2700 : Programming and Data Structures

Hashing : collision resolution

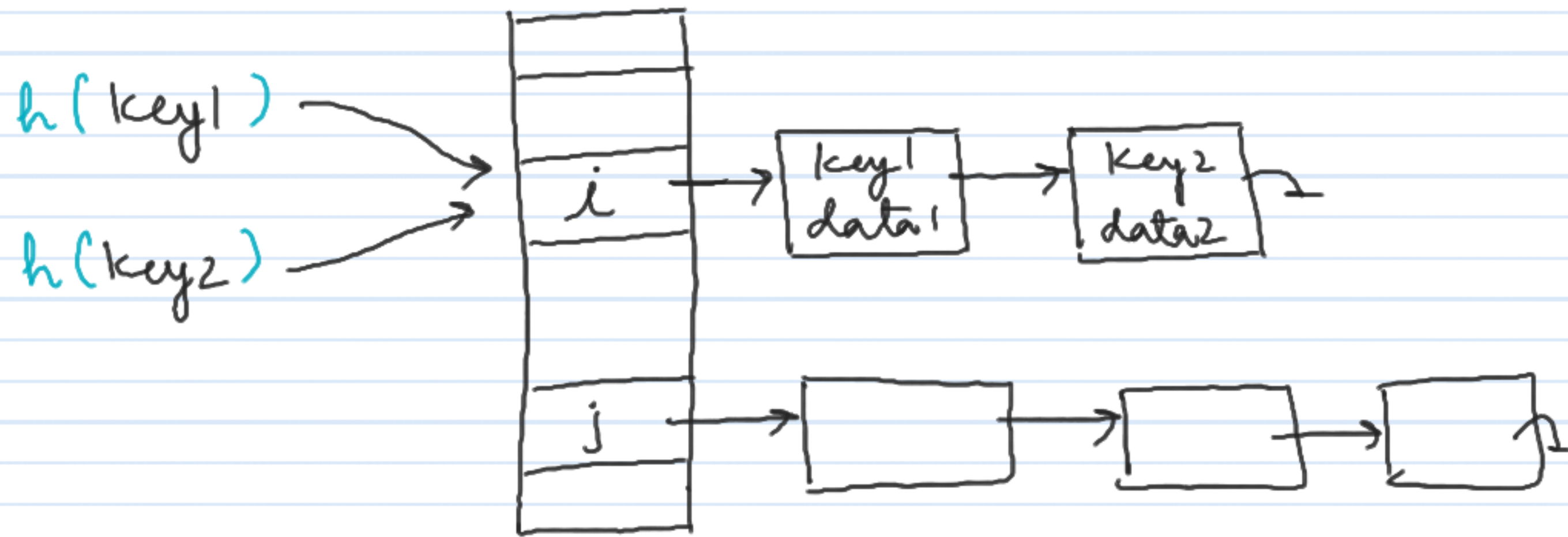
- Chaining
- Open Addressing
 - Linear Probing
 - Quadratic Probing
 - Double Hashing

Collision Resolution : Separate Chaining



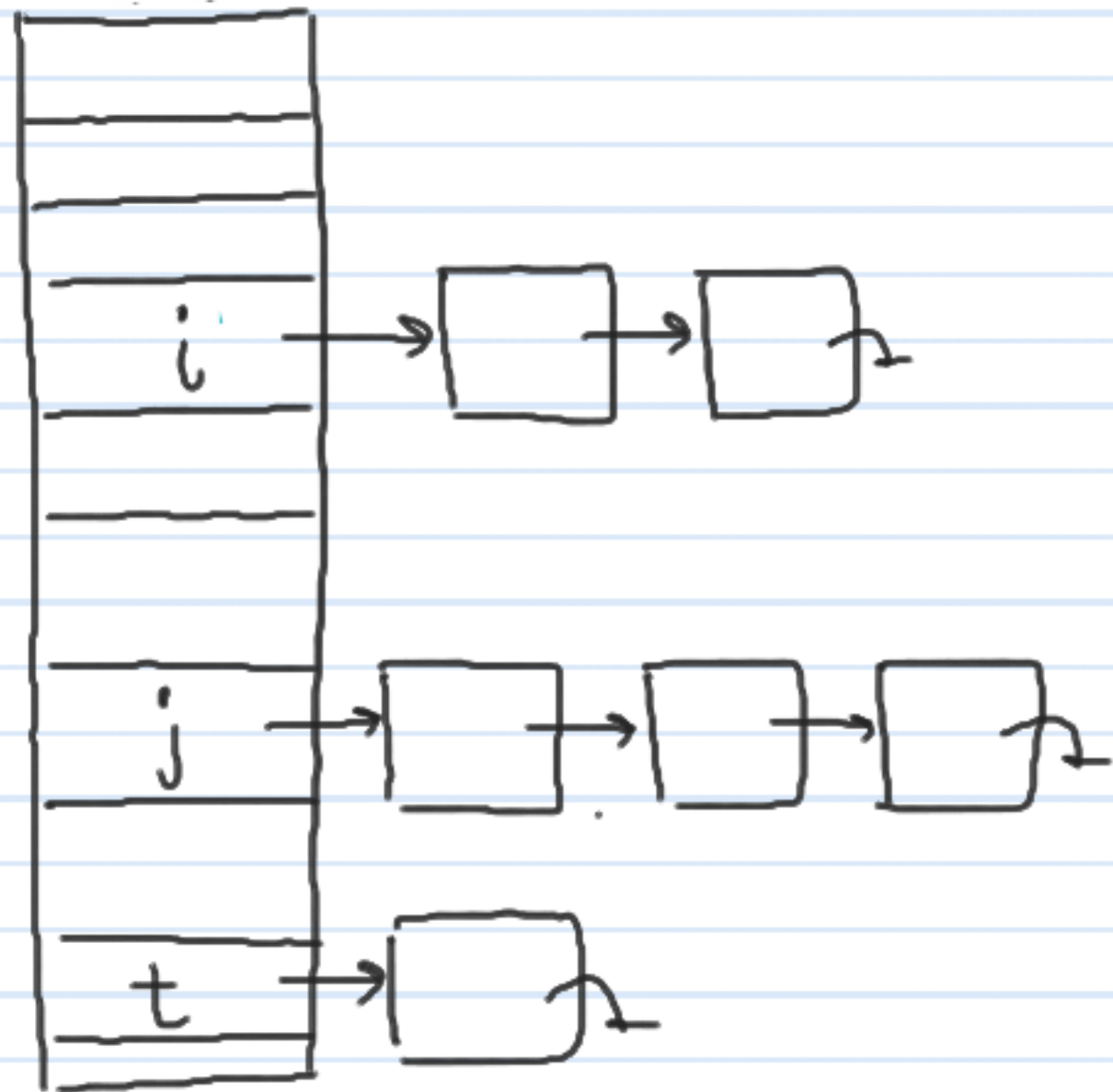
- We would like to store data corresp. to key_1 and key_2 (and possibly more keys) in the index i
 - all such keys have same hash value

Collision Resolution : Separate Chaining



- We would like to store data corresp. to key1 and key2 (and possibly more keys) in the index i
all such keys have same hash value

Collision Resolution : Separate Chaining



Insert (key, data)

$i = h(\text{key})$

insert into list $A[i]$

Search (key)

$i = h(\text{key})$

find in list $A[i]$

Delete (key)

$i = h(\text{key})$

delete from list $A[i]$

Note the importance of storing keys along with data.

DISADVANTAGES OF CHAINING?

LOAD FACTOR OF A HASH TABLE

n : # of keys in hash table

m : size of hash table (# of slots)

α : LOAD FACTOR

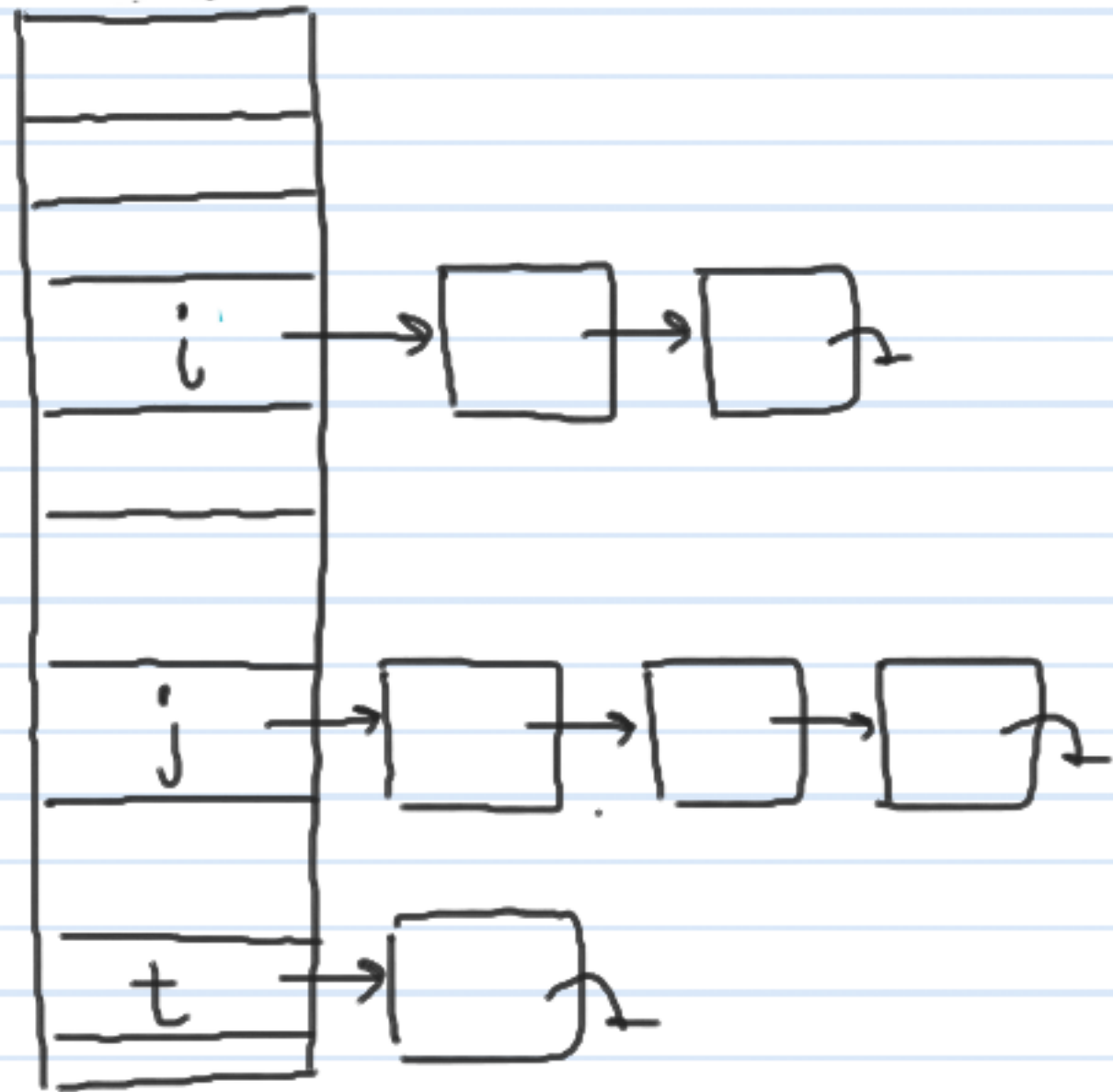
$$\alpha = n/m$$

Measure of how **full** the hash table is

In separate chaining α denotes average length of linked list

| caution : bad hash
| function can lead to
| large α

Collision Resolution : Separate Chaining



Insert (key, data)
 $i = h(\text{key})$
insert into list $A[i]$ } $O(1)$

Search (key)
 $i = h(\text{key})$
find in list $A[i]$ } $O(1 + \alpha)$

Delete (key)
 $i = h(\text{key})$
delete from list $A[i]$ } $O(1 + \alpha)$

If α is bounded by a constant average time is constant, **note that worst case may still be bad!**

Collision Resolution : OPEN ADDRESSING.

- Chaining needs two data structures

(array, aux DS for handling collisions)

↙
this DS can be list, BST, or even a hash table again!

- Typically needs pointers and cache inefficient.
- Can we do everything within the array?

Collision Resolution : OPEN ADDRESSING.

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↙
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Collision Resolution : OPEN ADDRESSING.

- In chaining hash value for a key is **fixed**
(not open)
- When hash value can be "open" (not fixed)
it is called open addressing.

How to search with "open" hash values

OPEN ADDRESSING :

- Hashed items reside in the array itself
- Hash value gives an index (home address)
- Multiple **probes** may be needed to find key.

Hash table

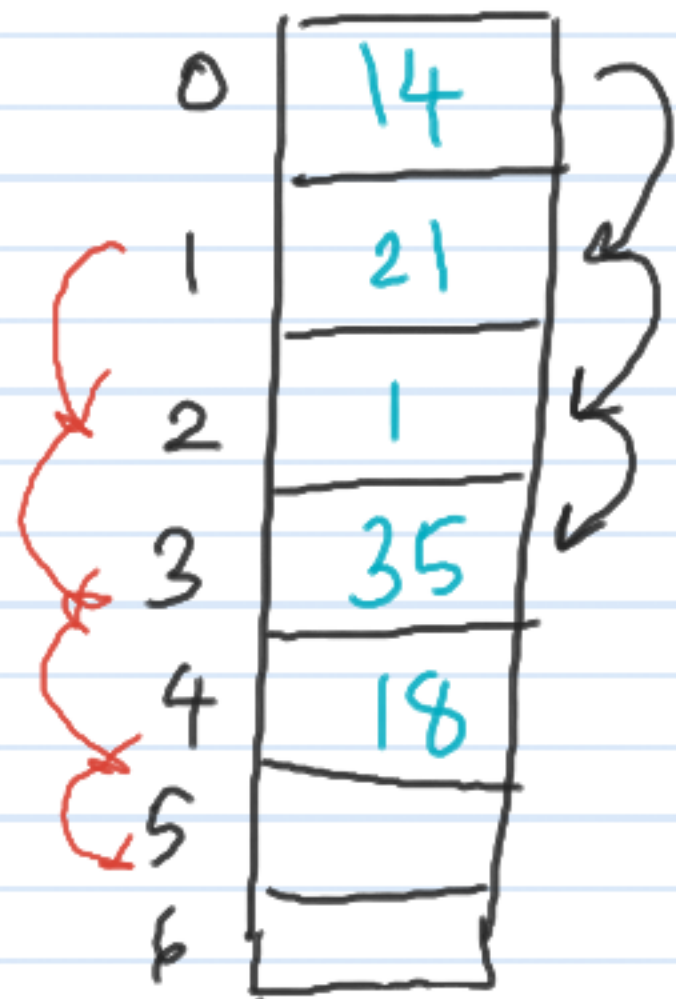


OPEN ADDRESSING : Linear Probing.

When collision occurs scan forward for available empty slot

Example :

$$h(\text{key}) = \text{key} \% 7;$$



insert 18, 14, 21, 1, 35,

find (35)

how many probes are needed?

find (8)

when to stop? how many probes?

OPEN ADDRESSING : Linear Probing.

When collision occurs scan forward for available empty slot

Example :

$$h(\text{key}) = \text{key} \% 7;$$

0	14
1	21
2	1
3	35
4	18
5	
6	

insert 18, 14, 21, 1, 35,

delete (21)

find (35)

↳ if our stopping criteria is empty slot there is an issue!

14
1
35
18

OPEN ADDRESSING : Linear Probing.

Deletion : lazy deletion

↳ simply mark the entry as deleted

Example :

3 states for every slot :

- occupied

- deleted

- empty

needs table size

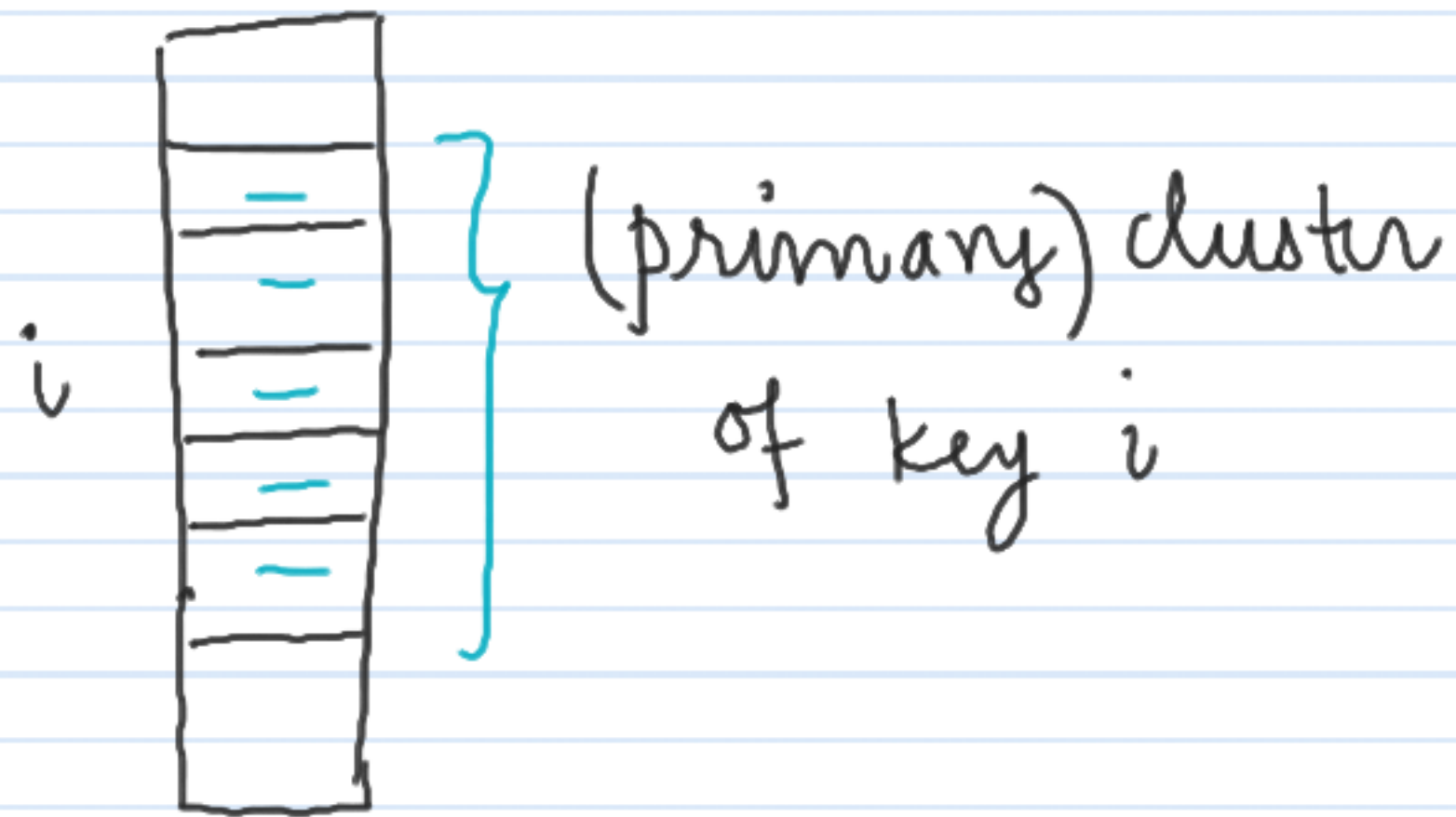
aux space.

occ	0	14
del	1	21
occ	2	1
occ	3	35
occ	4	18
em	5	
em	6	

deleted and empty are treated diff
by insert and find.

OPEN ADDRESSING : Linear probing (issue of clustering)

CLUSTER : A set of consecutive locations that are occupied.



- Large clusters slow down search and insert times.

- Clusters of 2 keys can merge forming an even larger cluster.

Can we leave gaps?

OPEN ADDRESSING : QUADRATIC PROBING.

Collision resolution function is quadratic

Linear probing : $h(\text{key}), h(\text{key})+1, h(\text{key})+2, \dots$

Quadratic probing : $h(\text{key}), h(\text{key})+1^2, h(\text{key})+2^2,$
 $h(\text{key})+3^2, \dots$

Example : $h(\text{key}) = \text{key} \% 7$

Insert 3, 18, 4, 38

0	38
1	
2	
3	3
4	18
5	4
6	

OPEN ADDRESSING : QUADRATIC PROBING.

collision resolution function is quadratic

Linear probing : $h(\text{key}), h(\text{key})+1, h(\text{key})+2, \dots$

Quadratic probing : $h(\text{key}), h(\text{key})+1^2, h(\text{key})+2^2,$
 $h(\text{key})+3^2, \dots$

Example : $h(\text{key}) = \text{key} \% 7$

Insert 3, 18, 4, 38

probes : 3, 3+1, $3+4=0$
successful.

Insert 12, 10 → what happens?

0	38
1	
2	
3	3
4	18
5	4
6	12

OPEN ADDRESSING : QUADRATIC PROBING.

Even if table is not full, our probes may not find a slot!

- does our probe finding terminate always? - no in general (even when table has slots)

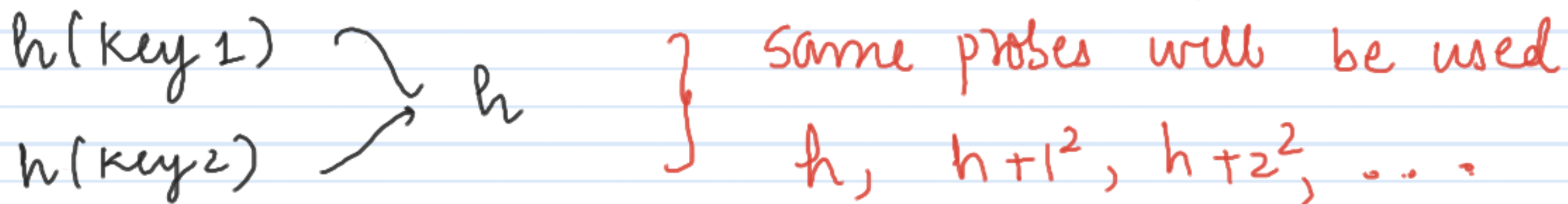
Claim: If Table size is an odd prime > 3 , and table is at least half empty, then quadratic probing is guaranteed to terminate.

why?

$$h(\text{key}) + i^2 = h(\text{key}) + j^2$$

OPEN ADDRESSING : QUADRATIC PROBING.

SECONDARY CLUSTERS.



These are called secondary clusters

↳ analogue of primary clusters in linear probing.

Can we generate different probe sequences?