

# CS 2700 : PROGRAMMING & DATA STRUCTURES.

## PRIORITY QUEUE ADT.

### GOALS:

- WHAT IS A PRIORITY QUEUE?
- IMPLEMENTATION
- APPLICATION

# PRIORITY QUEUE.

## Printer Queue

J <sub>1</sub>	J <sub>2</sub>	J <sub>3</sub>	J <sub>4</sub>	J <sub>5</sub>	J <sub>6</sub>	J <sub>7</sub>	J <sub>8</sub>
(1)	(1)	(100)	(2)	(2)	(1)	(4)	(1)

- Jobs are processed in queue order.  
(FIFO)

## In Practice

J<sub>3</sub> : book print

J<sub>6</sub> : flight ticket.

## PRIORITY QUEUE.

- Each element has a priority assigned  
part of input
- Assumptions :
  - priority values are distinct
  - lower value  $\Rightarrow$  higher priority

## PRIORITY QUEUE. (ADT)

- Each element has a priority assigned  
part of input

### Operations supported:

- Insert (enqueue)
- Delete Min (dequeue)  
↳ remove element  
with highest priority.

## PRIORITY QUEUE. (ADT)

- Each element has a priority assigned  
part of input
- $\langle \text{key}, \text{value} \rangle$  pair is inserted  
↳ priority
- We will insert only keys (illustration)
- retrieve element with highest priority  
(and delete it)

## PRIORITY QUEUE

I 10, I 20, I 5, I 6, I 13

D, D, I 2, I 12, D

Write down output of "D" operations

D : 5

D : 6

D : 2



# PRIORITY QUEUE (IMPLEMENTATIONS)

- LINKED LIST

$O(1)$

$O(n)$

- SORTED LINKED LIST

$O(n)$

$O(1)$

- BINARY SEARCH TREE

$O(n)$

$O(n)$

- AVL TREE

$O(\log n)$

$O(\log n)$

INSERT

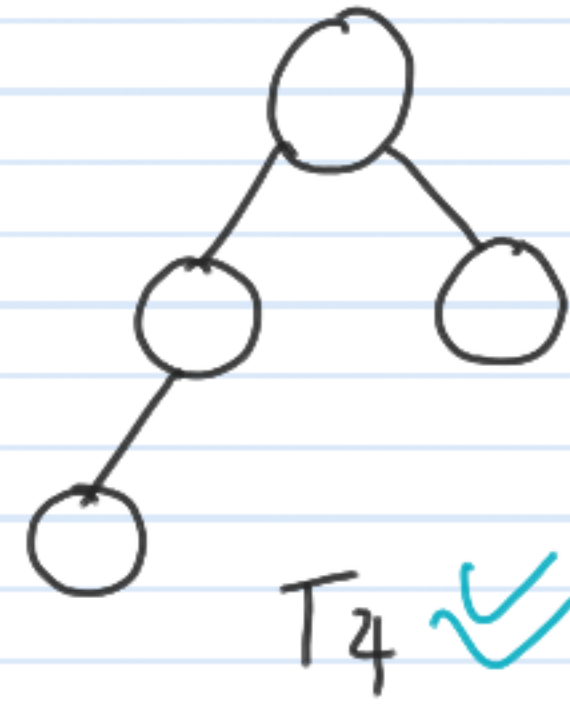
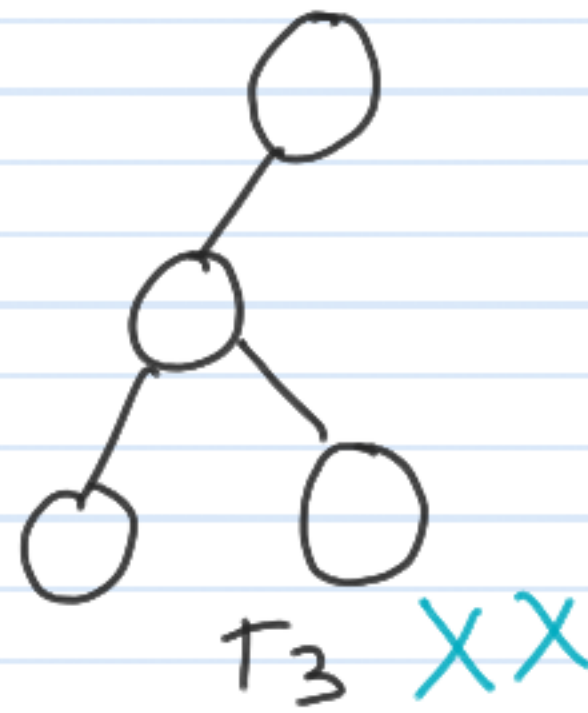
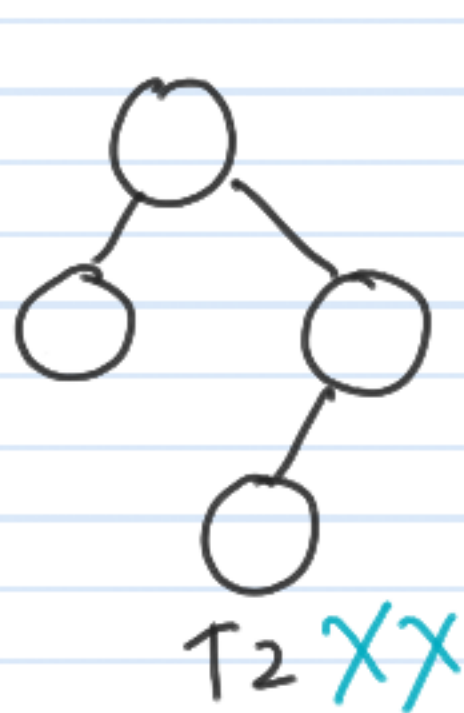
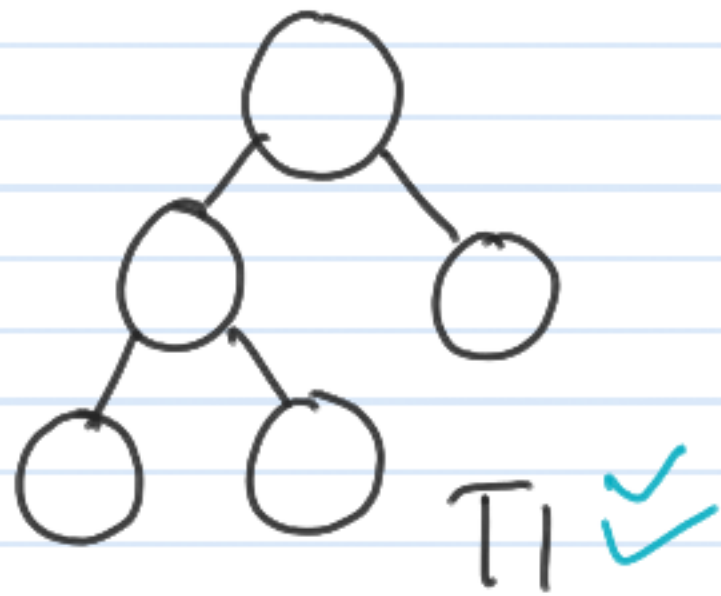
DELETE

MIN.

# PRIORITY QUEUE (IMPLEMENTATION)

Binary heap : Special Binary Tree

Structure :



Heap order :

Parent key < child key



# PRIORITY QUEUE (IMPLEMENTATION)

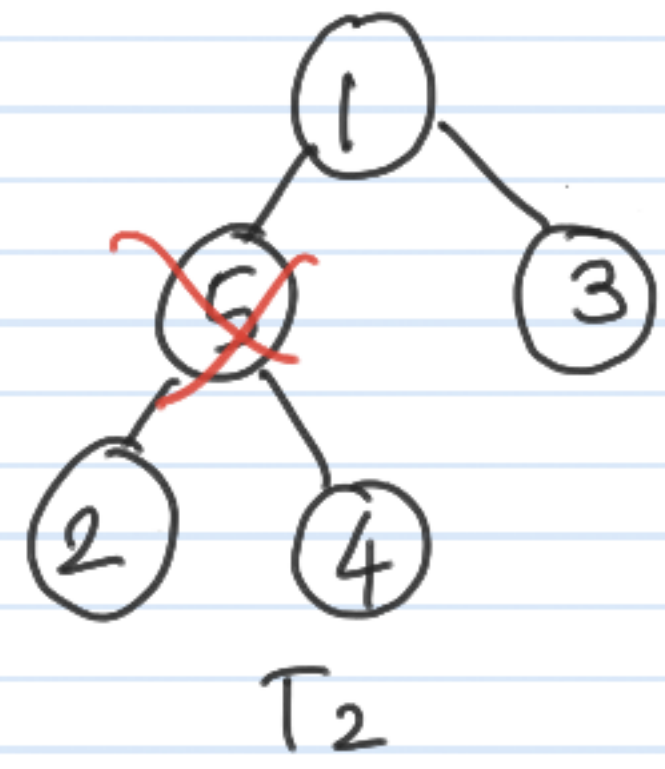
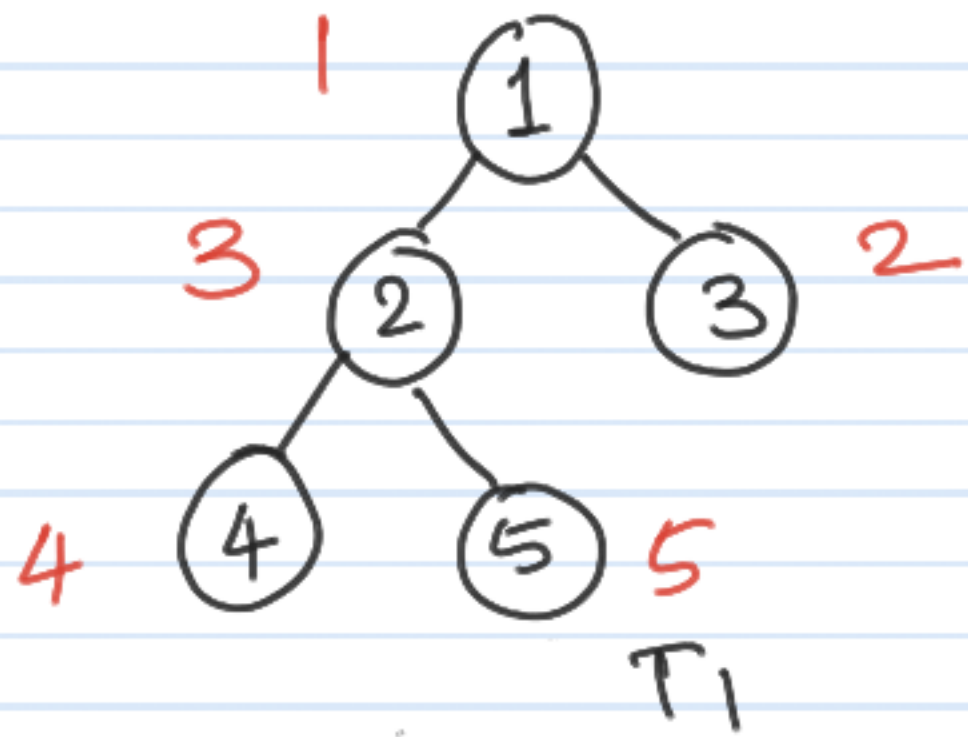
Binary heap : Special Binary Tree

- Complete binary Tree with possibly last level not full
- Last level : left to right
- Heap order

1, 2, 3, 4, 5

1 2, 5, 3, 4

1 3 2 4 5



# PRIORITY QUEUE (IMPLEMENTATION)

Binary heap : Special Binary Tree

- Complete binary Tree with possibly last level not full

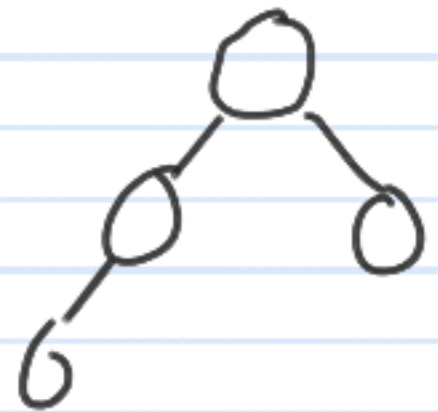
- Last level : left to right

- Heap order

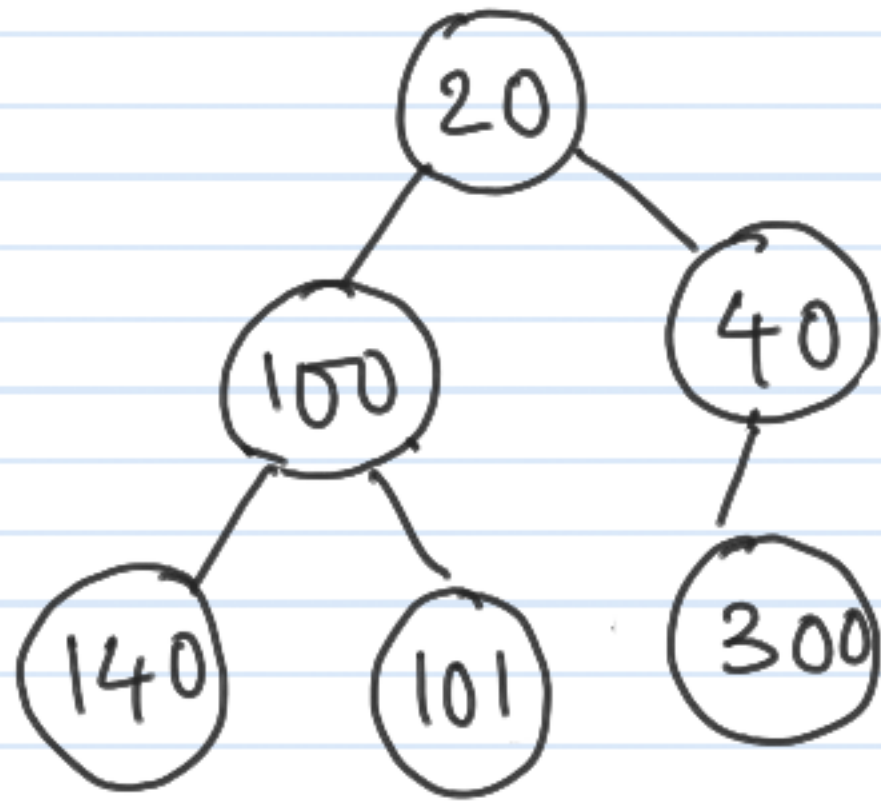
- # of nodes in a heap of height  $h$

$h=2 : 2^2 \dots 2^3 - 1 \quad [4, \dots 7]$   
 $h=3 : 2^3 \dots 2^4 - 1$

general case :  
 $(2^h) \dots (2^{h+1} - 1)$



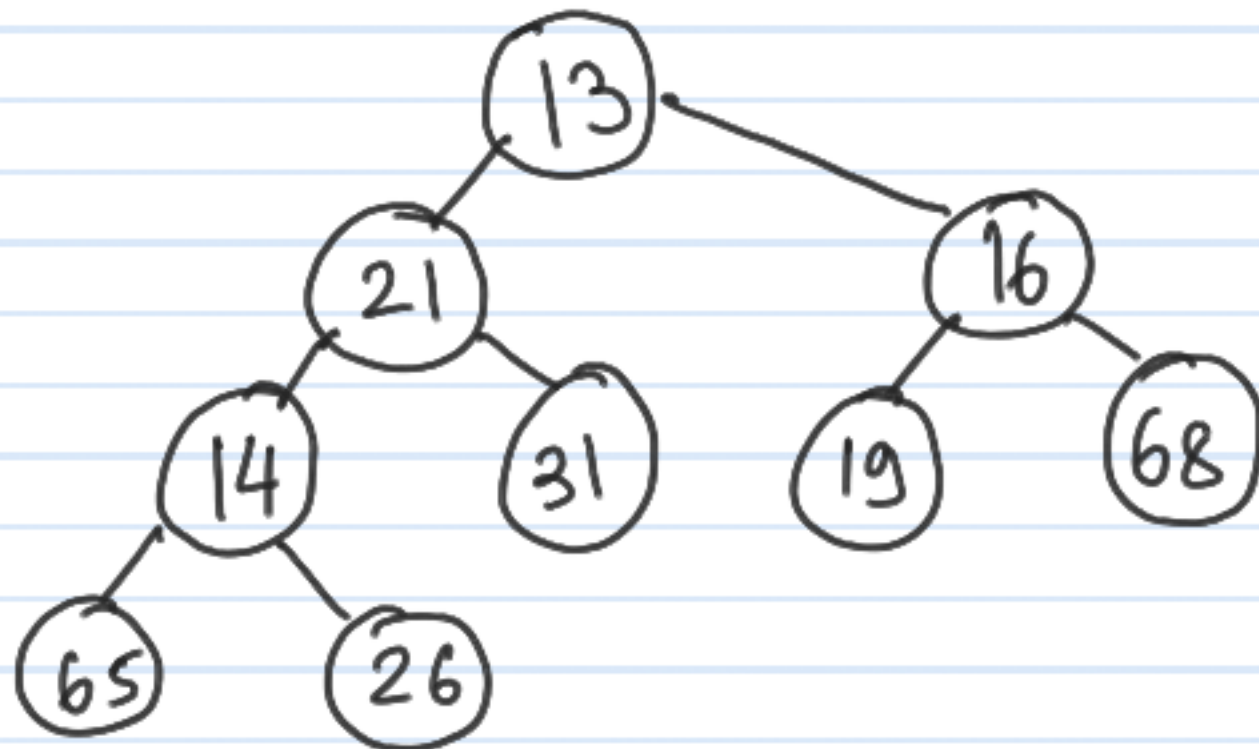
# BINARY HEAP (MIN Heap)



T<sub>1</sub>

Structure

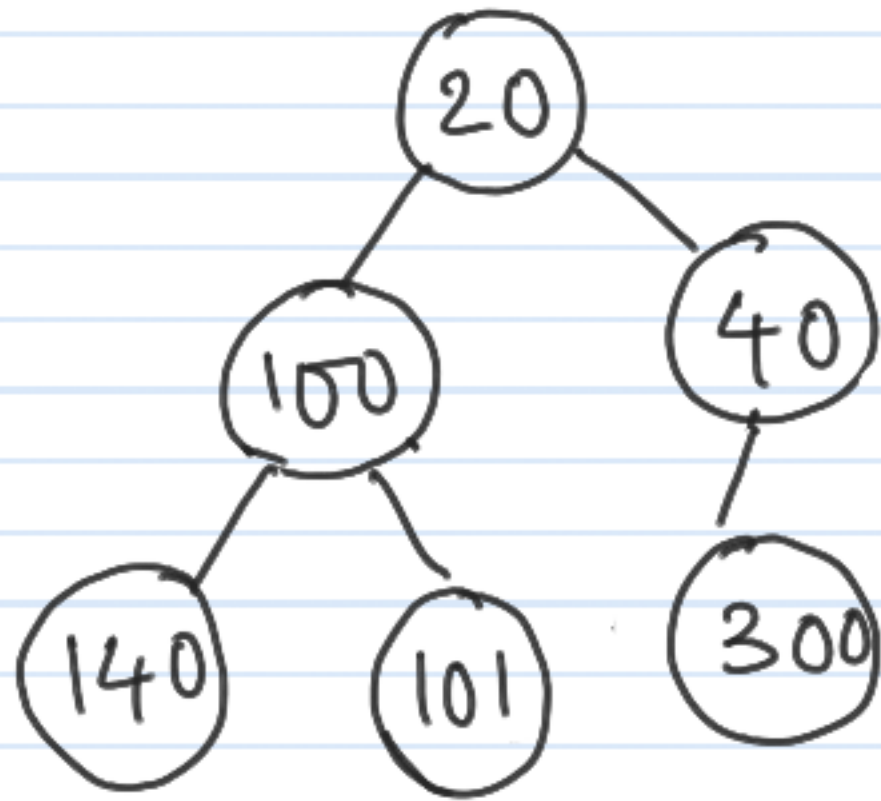
Heap order



T<sub>2</sub>

Heap Order vs  
BST property

# BINARY HEAP (MIN Heap)

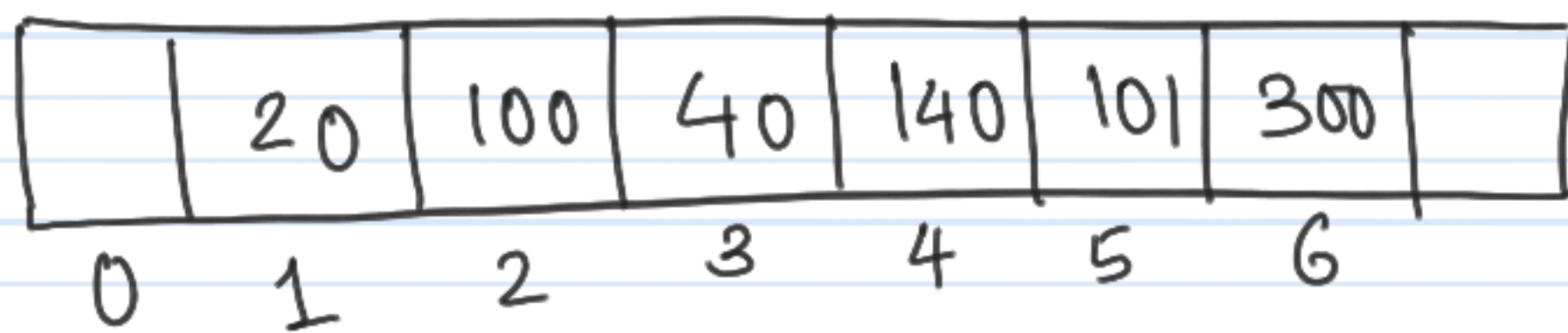


$T_1$

Structure

Heap order

- Conveniently represented as an **array**

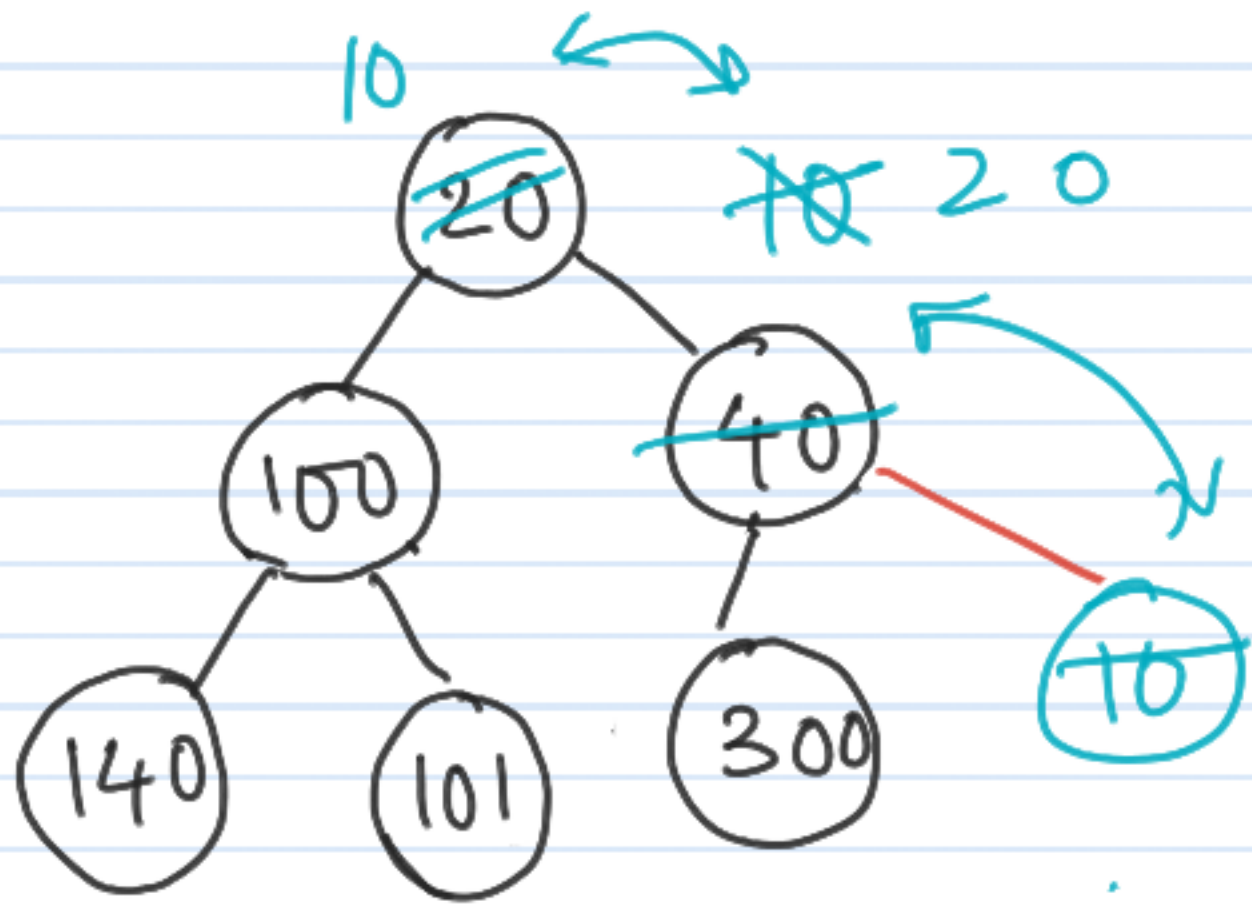


index  $i$

Parent :  
Children :



# BINARY HEAP (MIN Heap)

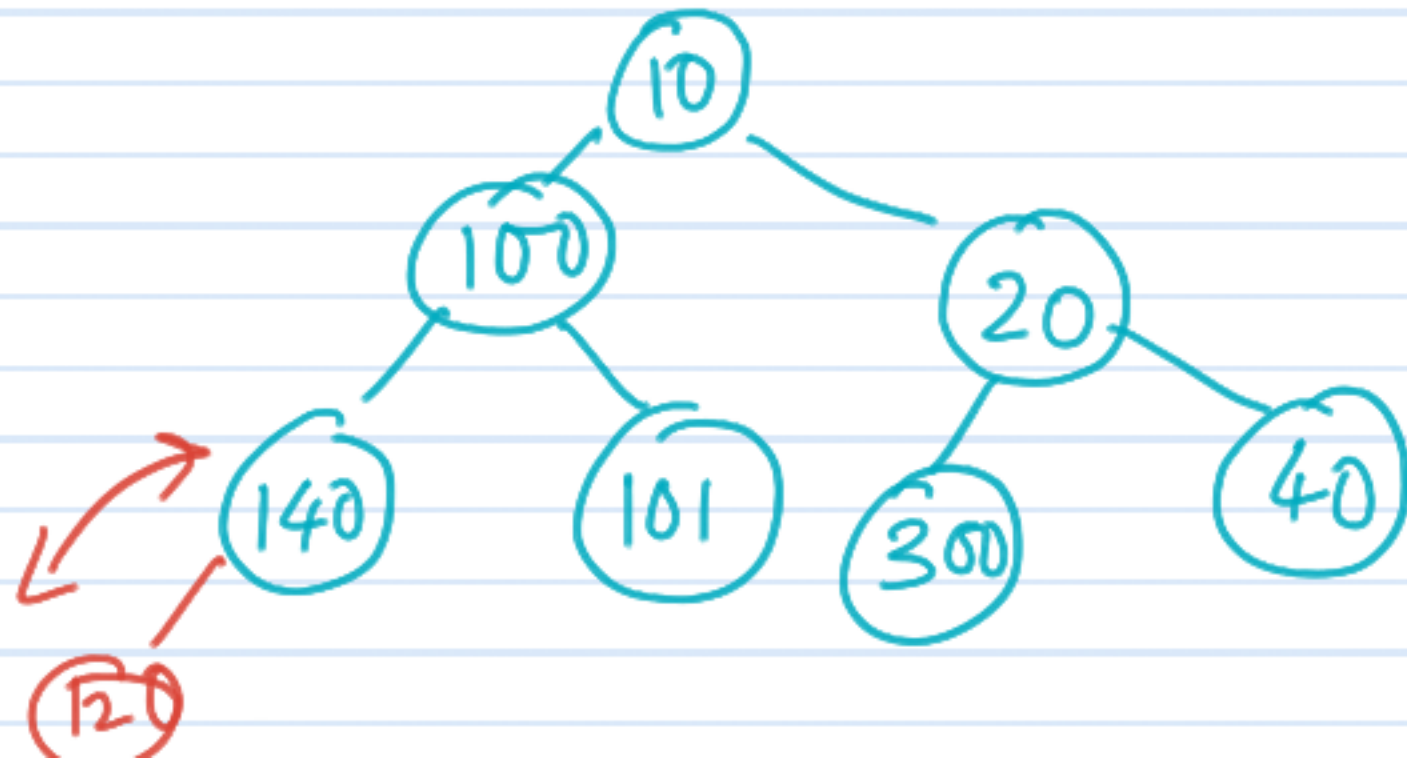
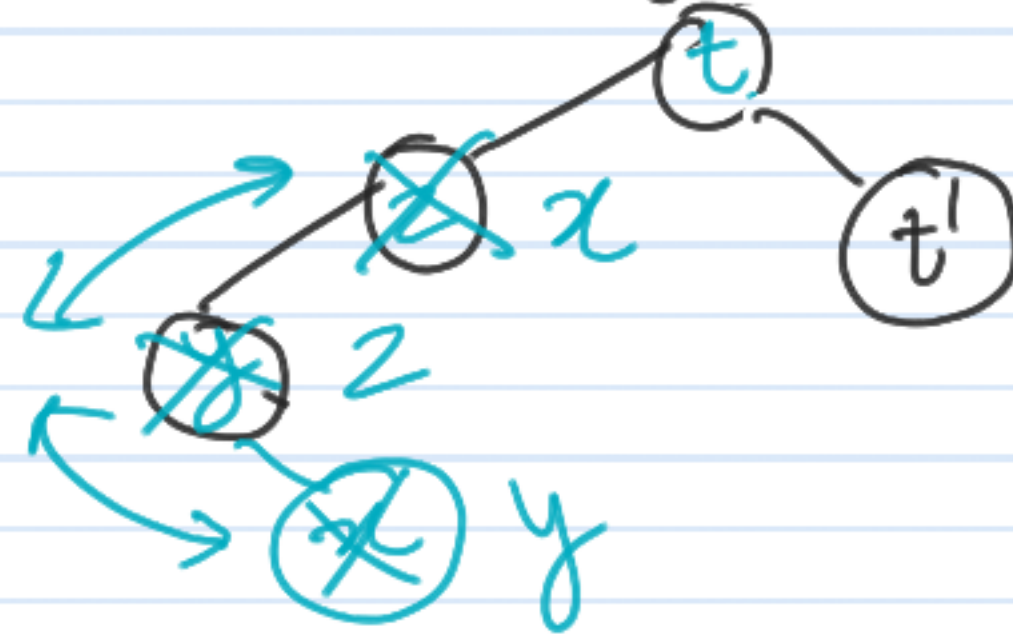


insert (120)

- Find next available hole (location)

- Restore heap order

if violated.

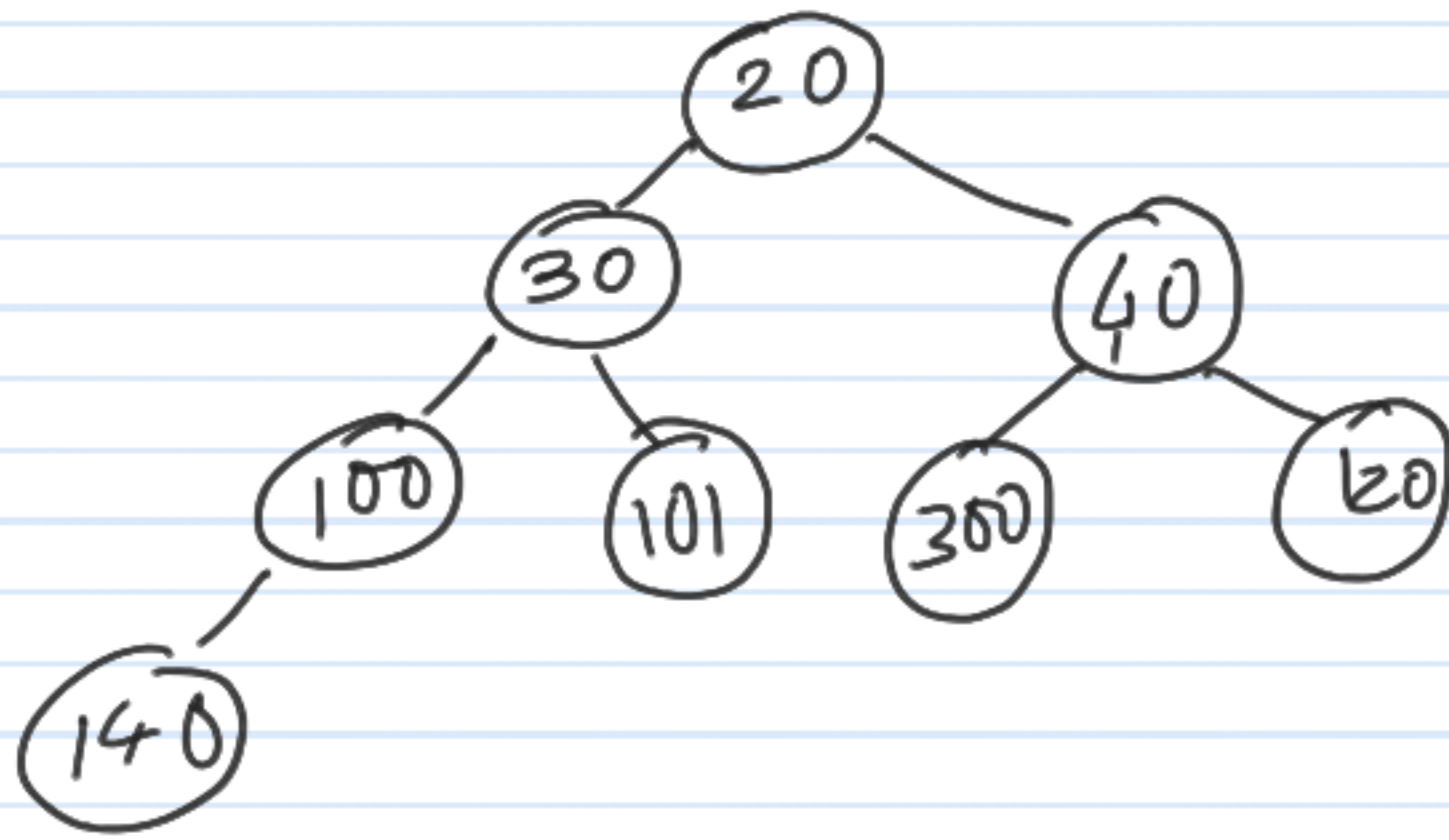
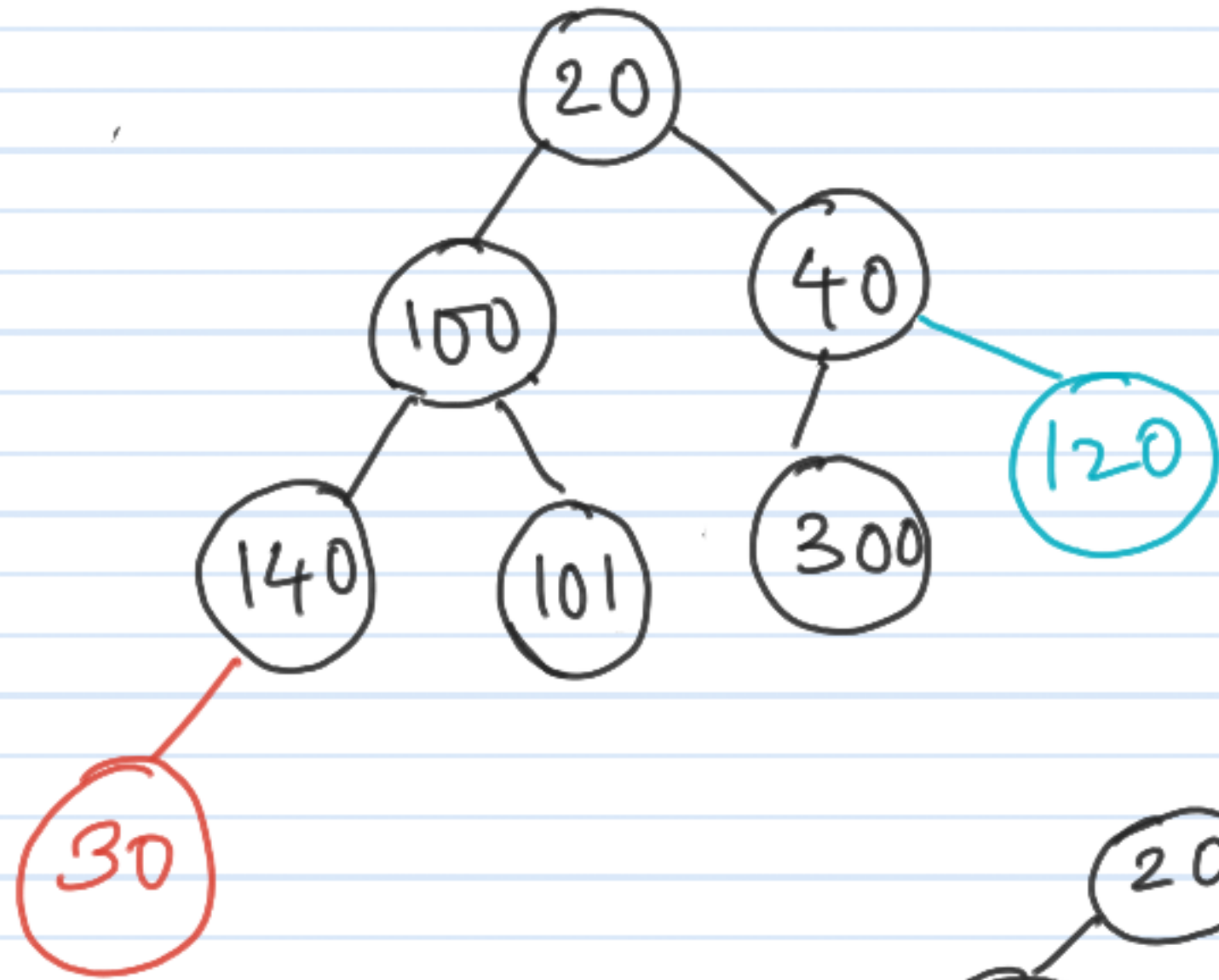




# BINARY HEAP (MIN Heap)

- insert (120)

- insert (30)

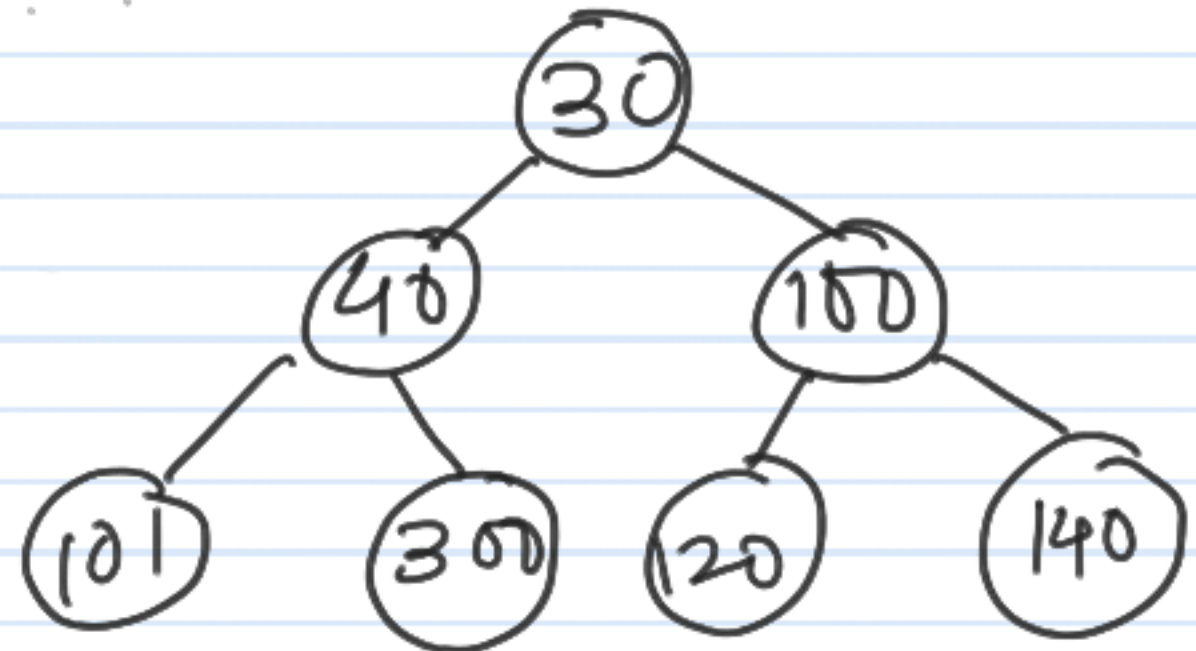
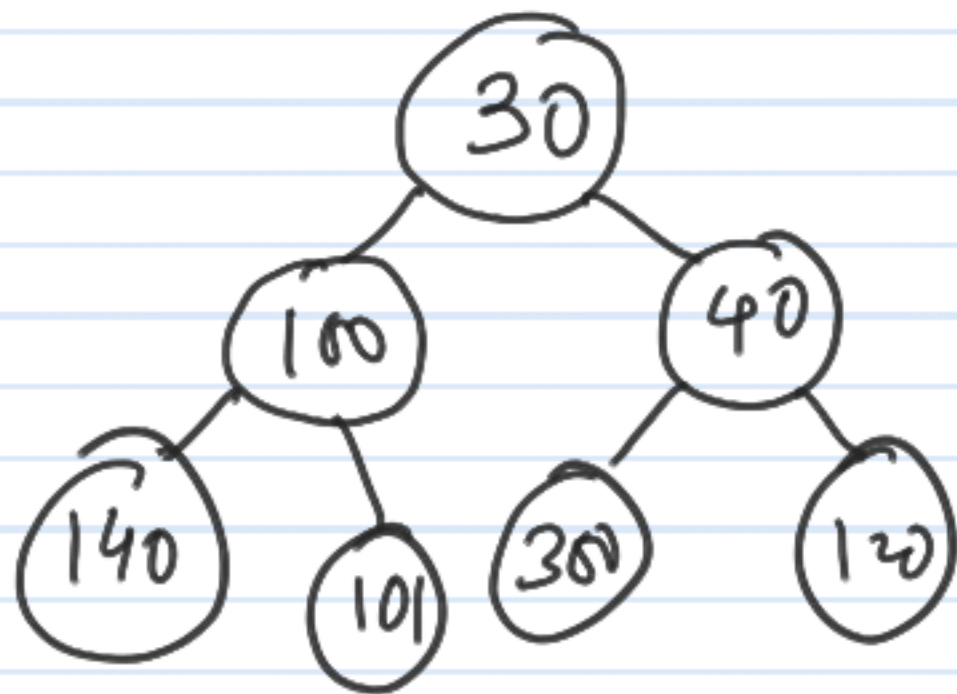
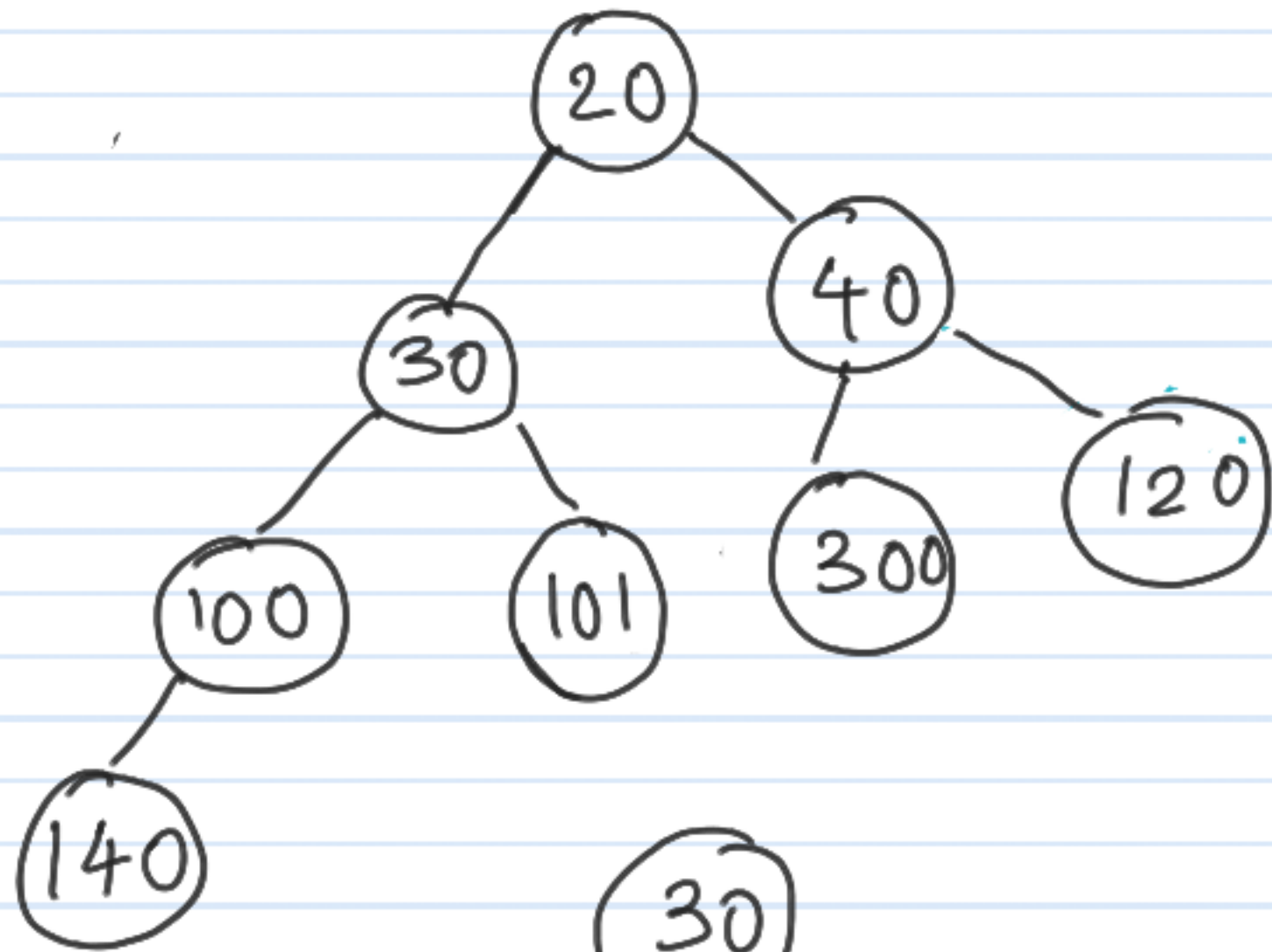


- Restore heap  
order

- element moves  
up  
(percolate up)

# BINARY HEAP (MIN Heap)

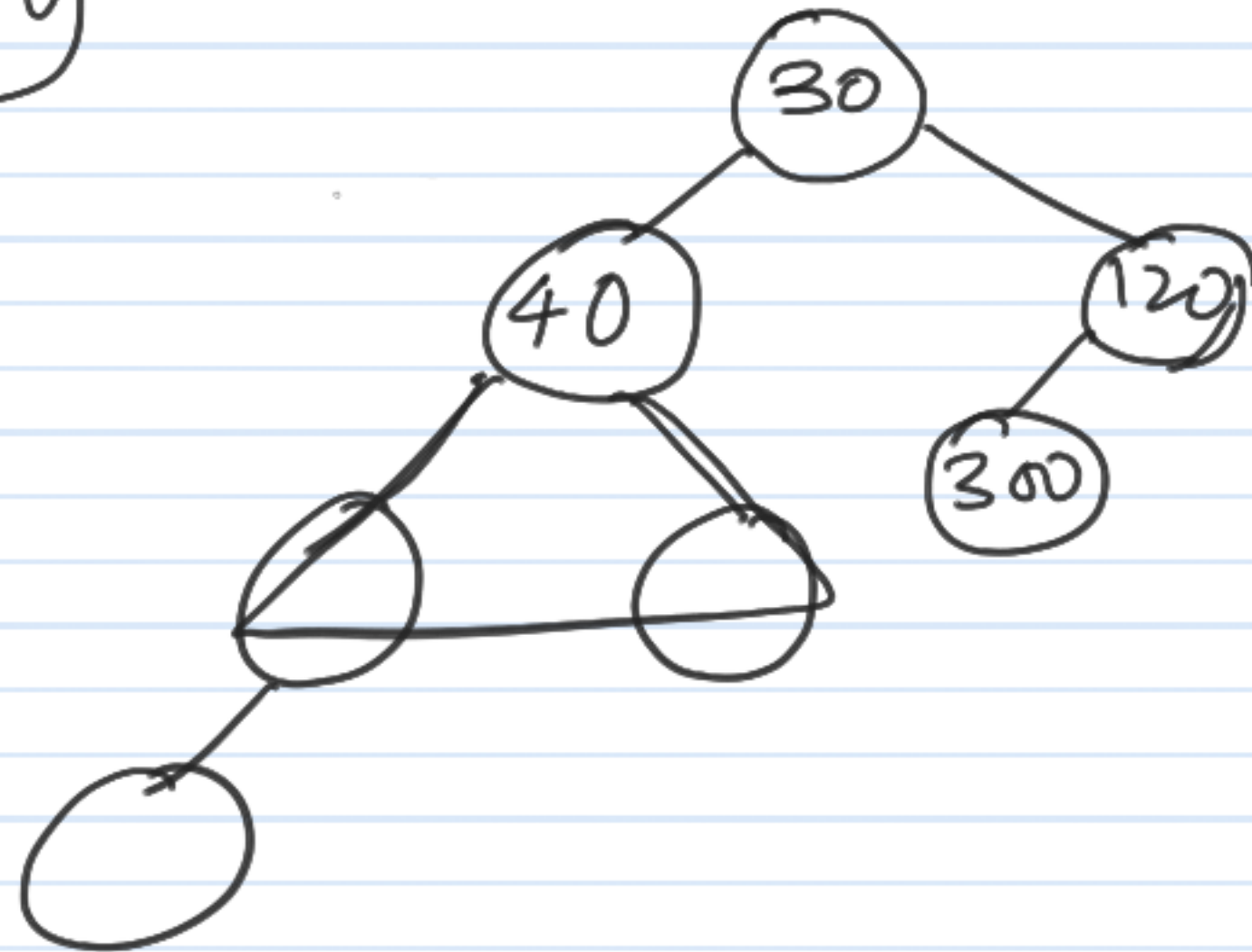
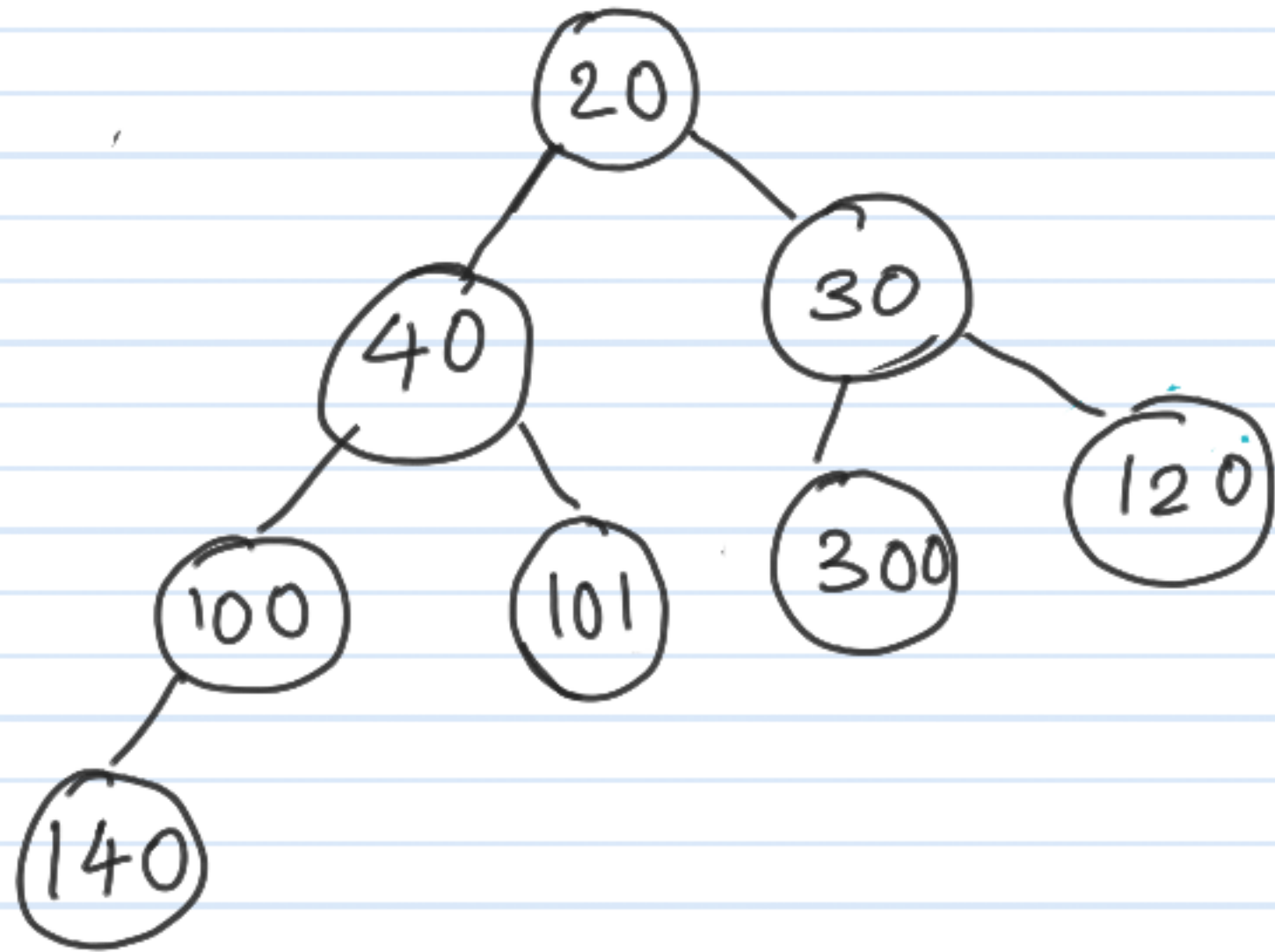
- Delete Min



Shape after deletion

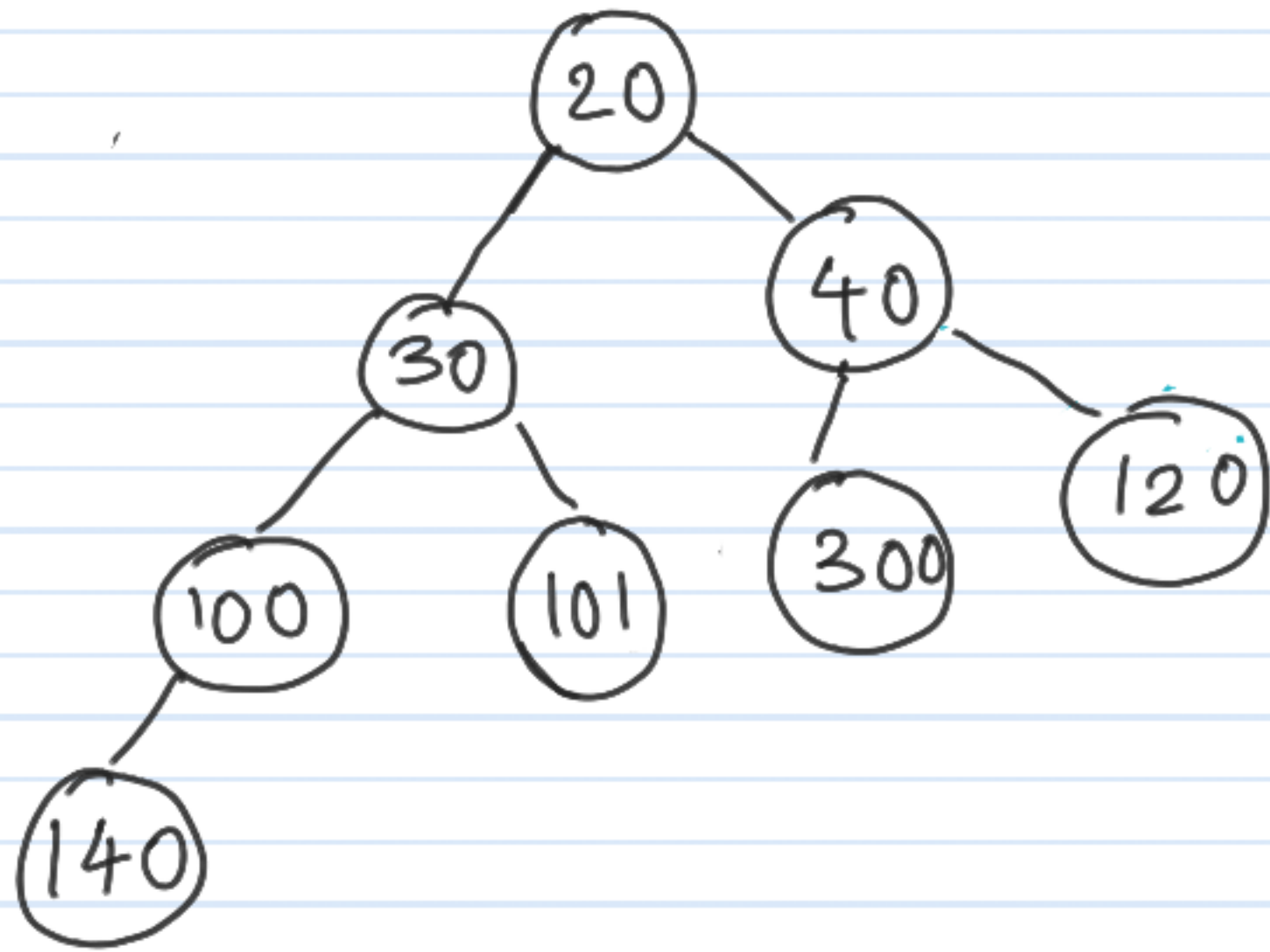
# BINARY HEAP (MIN Heap)

- Delete Min

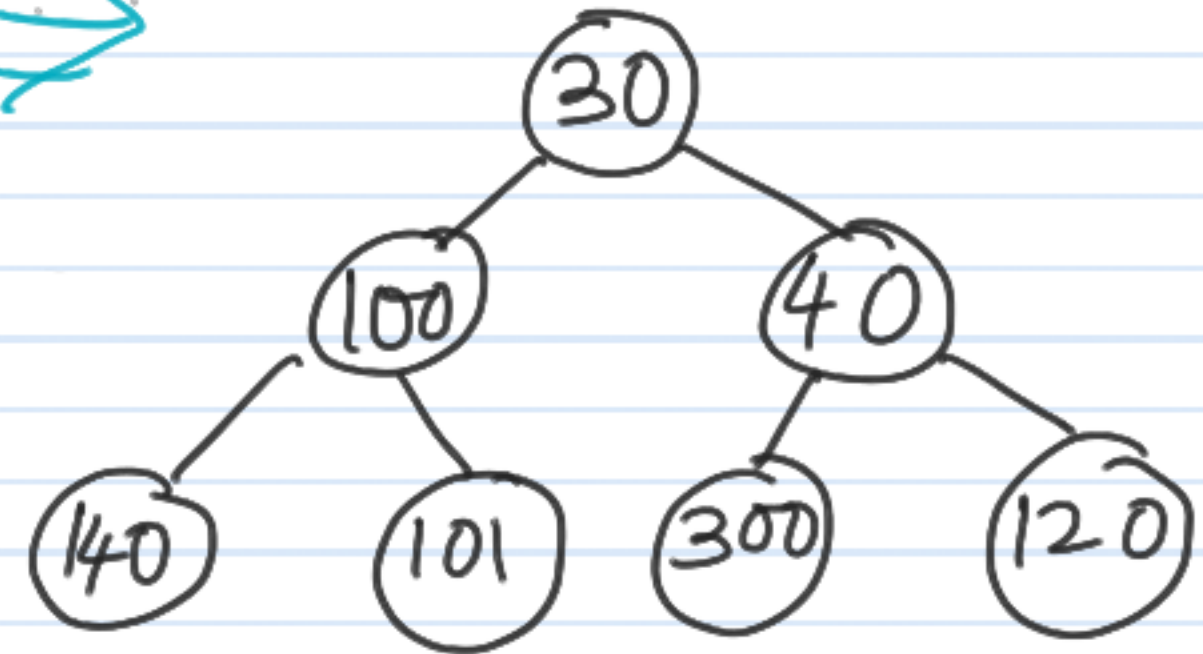
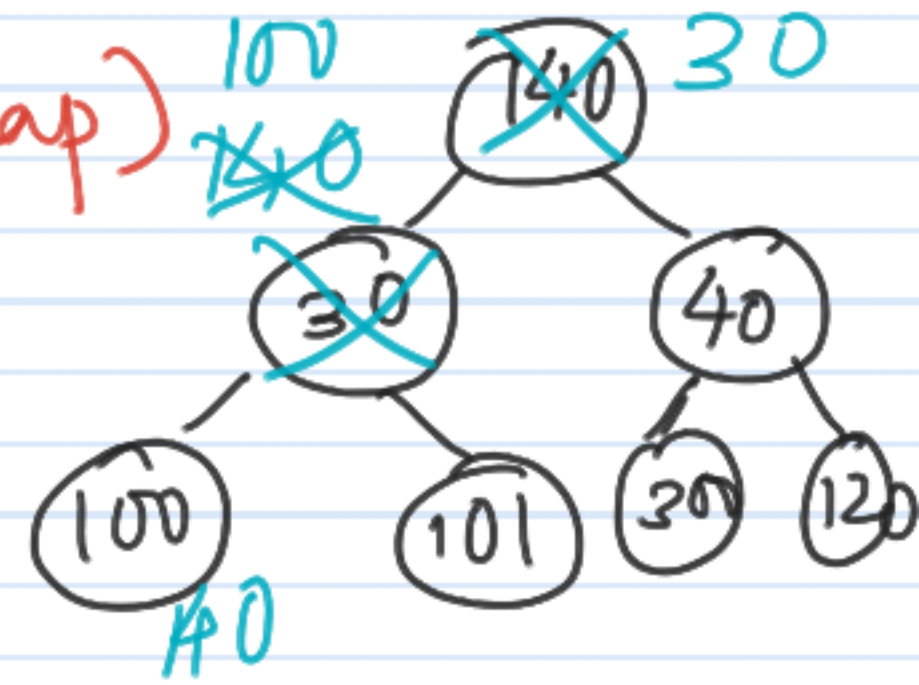


# BINARY HEAP

(MIN Heap)



- Delete Min

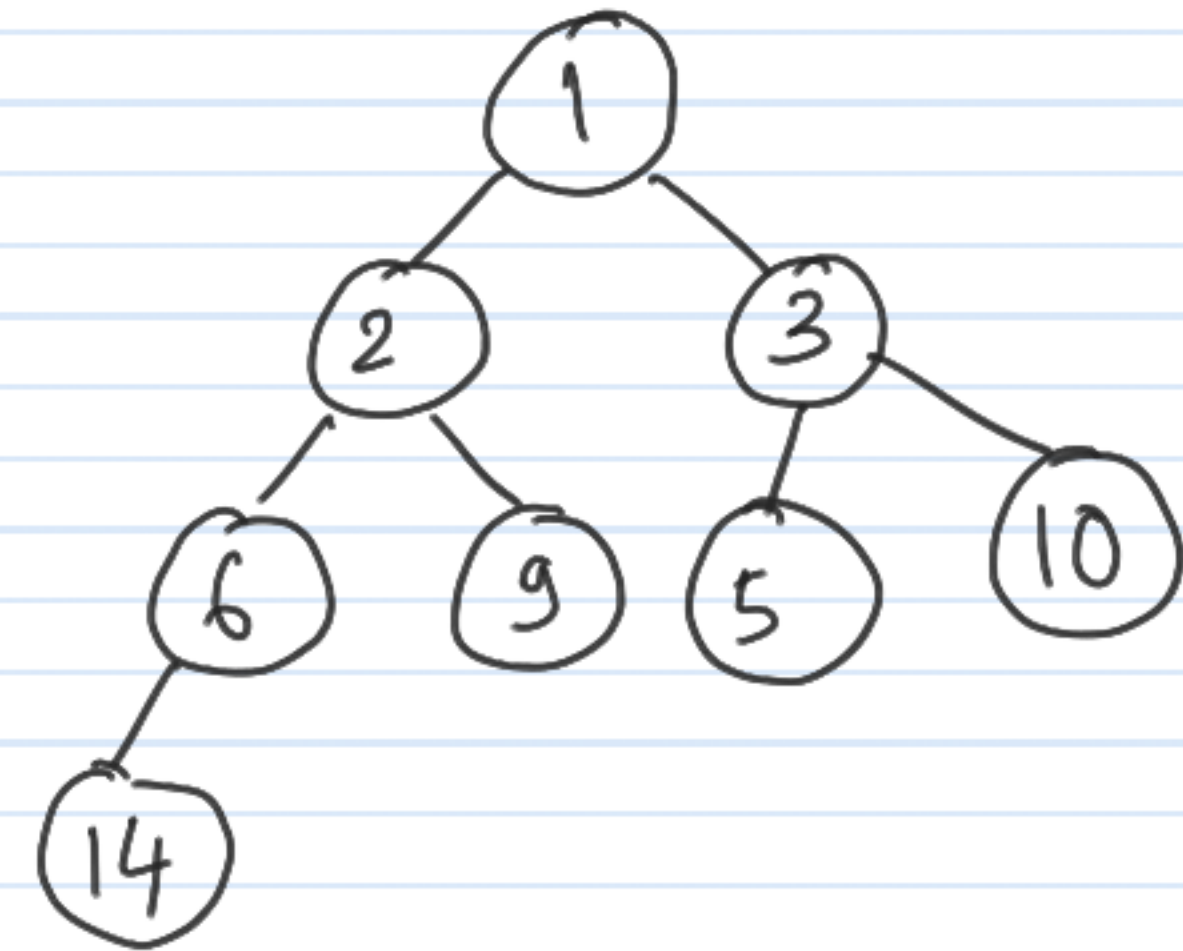


Shape after deletion

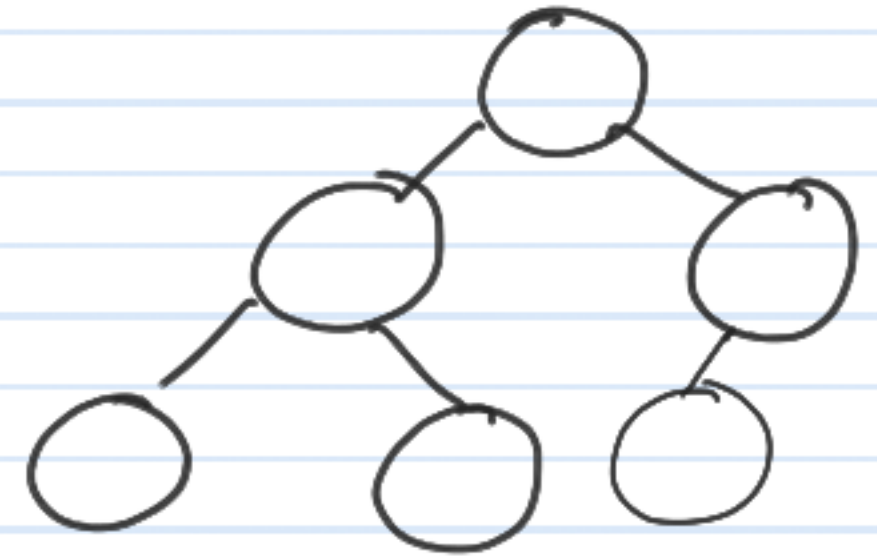
- Remove element in root
- Replace root with last elem.
- Percolate down.



# BINARY HEAP : EXAMPLE



delete min  
insert(13)  
delete min  
delete min

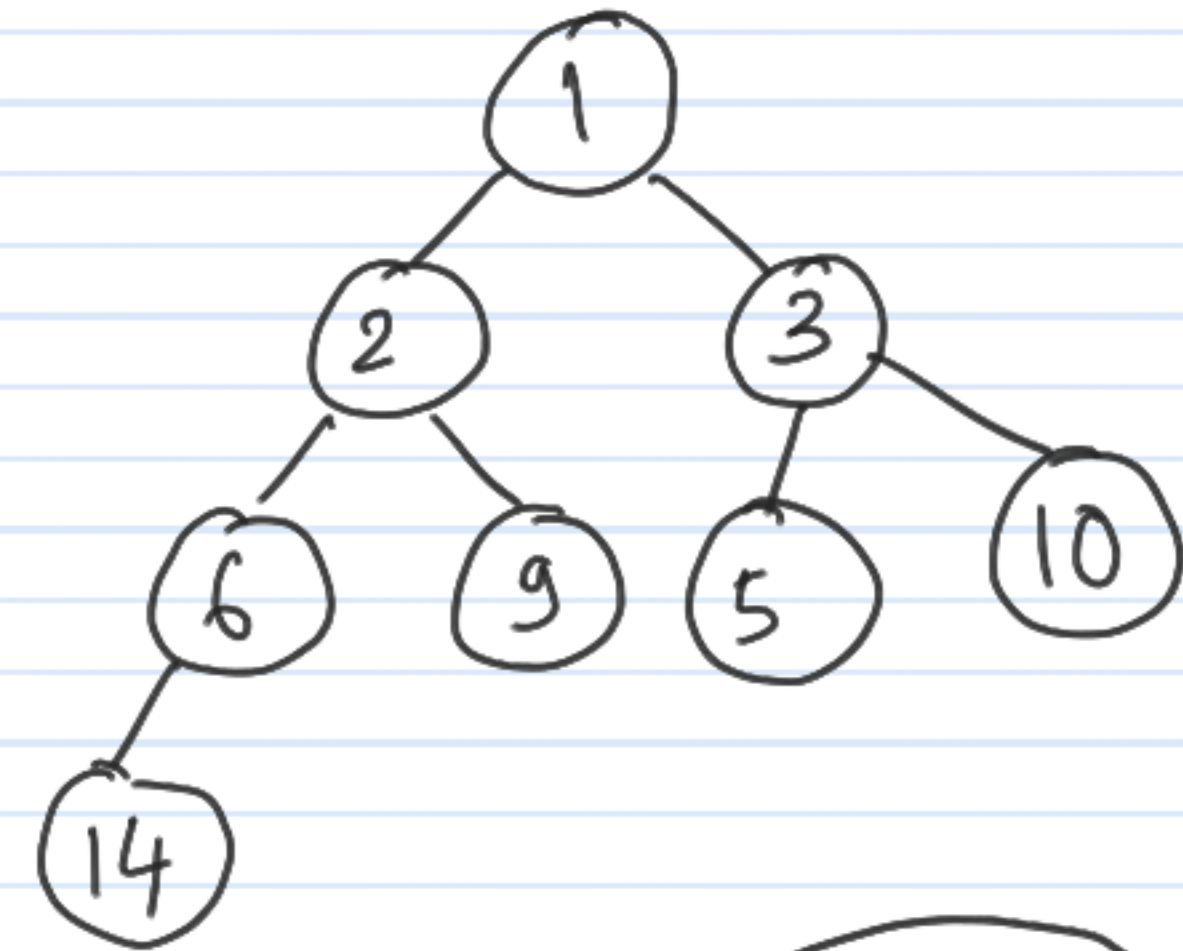


3, 6, 10, 13, 9, 14

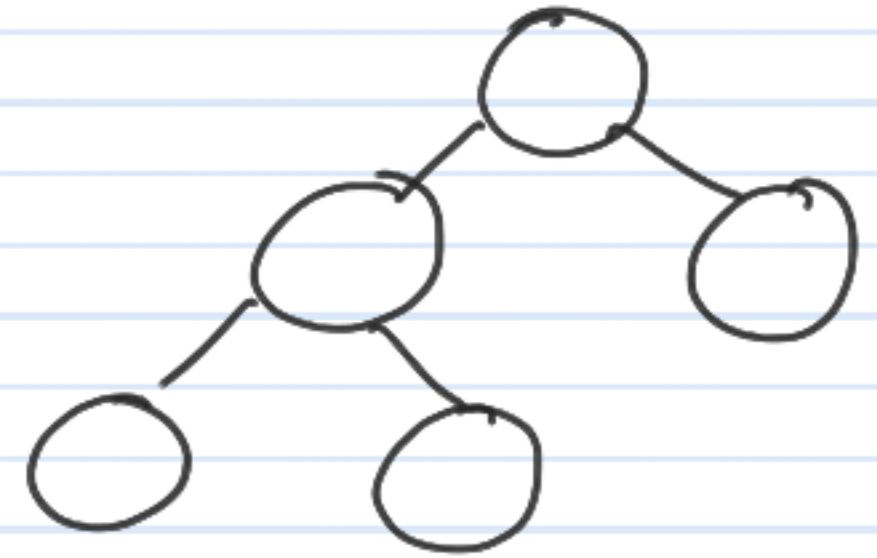
6, 9, 10, 13, 14, 9



# BINARY HEAP : IMPLEMENTATION



delete min  
insert(13)  
delete min  
delete min



insert (e)

